1. $\displaystyle\int_1^4 \left(\frac{3\sqrt{x}-5}{\sqrt{x}}\right) dx =$

(a) -1

(b) $\dfrac{1}{2}$

(c) $-\dfrac{3}{2}$

(d) $\dfrac{5}{2}$

(e) $-\dfrac{1}{2}$

2. If $f(x) = \begin{cases} 3^x & 0 \le x \le 1 \\ 3x^2+1 & 1 < x \le 2 \end{cases}$, then $\displaystyle\int_0^2 f(x)dx =$

(a) $8 + \dfrac{2}{\ln 3}$

(b) $8 + \dfrac{3}{\ln 3}$

(c) $10 + \dfrac{1}{\ln 3}$

(d) $8 - \dfrac{2}{\ln 3}$

(e) $6 + \dfrac{4}{\ln 3}$

3. The Riemann sum for $f(x) = \dfrac{15}{x}$, $1 \leq x \leq 3$ with four subintervals, taking the sample points to be right endpoints, is equal to

(a) $\dfrac{57}{4}$

(b) $\dfrac{29}{4}$

(c) $\dfrac{63}{4}$

(d) $\dfrac{59}{8}$

(e) $\dfrac{63}{8}$

4. $\displaystyle\int \frac{\cos\theta + \cos\theta\cot^2\theta}{\csc^2\theta} d\theta =$

(a) $\sin\theta + c$

(b) $\tan\theta + c$

(c) $\cot\theta + c$

(d) $\cos\theta + c$

(e) $\csc\theta + c$

5. The limit $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{4n}\tan\left(\frac{\pi i}{4n}+\frac{\pi}{3}\right)$ can be interpreted as the area under the graph of the function

(a) $y=\tan\left(x+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(b) $y=\tan\frac{1}{4}\left(x+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(c) $y=\tan\left(\frac{x}{4}+\frac{\pi}{3}\right),\quad 0\le x\le\frac{\pi}{4}$

(d) $y=\frac{\pi}{3}+\tan x,\quad 0\le x\le\frac{\pi}{4}$

(e) $y=\tan\left(x+\frac{\pi}{3}\right),\quad \frac{\pi}{4}\le x\le\frac{\pi}{3}$

6. An expression for the area under the graph of $f(x)=4x-x^2,\ 2\le x\le 4,$ as a limit and using right endpoints is

(a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8}{n}-\frac{8}{n^3}i^2\right)$

(b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8}{n}-\frac{4}{n}i-\frac{8}{n^3}i^2\right)$

(c) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{12}{n}-\frac{16}{n^3}i^2\right)$

(d) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4}{n}-\frac{8}{n^3}-\frac{16}{n^3}i^2\right)$

(e) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8}{n}+\frac{16}{n^3}i^2\right)$

7. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n} + 2\right)^2 \frac{1}{n} =$

(a) $\frac{19}{3}$

(b) $\frac{29}{6}$

(c) $\frac{31}{3}$

(d) $\frac{19}{6}$

(e) $\frac{38}{3}$

8. The slope of the tangent line to the graph of the function $f(x) = \int_{\tan x}^{1} \frac{1}{\sqrt{1+t^2}}\, dt$ at $x = \frac{\pi}{3}$ is equal to

(a) -2

(b) $\frac{1}{2}$

(c) $-\sqrt{3}$

(d) $-\frac{1}{2}$

(e) 2

9. Which one of the following integrals **exists** according to the Fundamental Theorem of Calculus?

(a) $\displaystyle\int_0^{\pi/4} \cot\left(x + \frac{\pi}{2}\right)\, dx$

(b) $\displaystyle\int_0^{1} \ln x\, dx$

(c) $\displaystyle\int_{\pi/4}^{\pi} \sec x\, dx$

(d) $\displaystyle\int_{-5}^{5} \frac{2}{x^7}\, dx$

(e) $\displaystyle\int_0^{2} \frac{e^{x-1}}{(x-1)^2}\, dx$

10. If a particle moves along a line so that its velocity at time t is $v(t) = 3t - 3$ (measured in meters per second), then the distance (in meters), traveled by the particle during the time period $\frac{1}{2} \le t < 2$ is equal to

(a) $\frac{15}{8}$

(b) $\frac{9}{8}$

(c) $\frac{11}{4}$

(d) $\frac{13}{4}$

(e) $\frac{17}{8}$

11. $\displaystyle\int \frac{5}{\sqrt{x}(3\sqrt{x}+4)^{3/5}}\,dx =$

(a) $\dfrac{25}{3}(3\sqrt{x}+4)^{2/5}+c$

(b) $\dfrac{25}{6}x^{2/5}+\dfrac{25}{8}x^{1/2}+c$

(c) $\dfrac{1}{75}(3\sqrt{x}+4)^{2/5}+c$

(d) $\dfrac{25}{6}x^{2/5}+\dfrac{25}{16}x^{1/2}+c$

(e) $25(3\sqrt{x}+4)^{2/5}+c$

12. Which one of the following statements is **FALSE** about the function $f(x)=\displaystyle\int_1^x \frac{1}{t}\,dt, \quad x>0$?

(a) $f(x_1+x_2)=f(x_1)+f(x_2)$ for all $x_1, x_2>0$

(b) f is increasing for all $x>0$

(c) $\displaystyle\lim_{x\to 0^+} f(x)=-\infty$

(d) $e^{f(e)}=e$

(e) The graph of f is concave downward for all $x>0$.

13. The area enclosed by the line $2x + y = 1$ and the parabola $y = 4 - x^2$ is given by the definite integral

(a) $\int_{-1}^{3} (3 + 2x - x^2)\, dx$

(b) $\int_{-5}^{3} \left(\frac{1}{2}(1 - y) - \sqrt{4 - y} \right)\, dy$

(c) $\int_{-1}^{3} (x^2 - 2x - 3)\, dx$

(d) $\int_{-5}^{4} \left(\frac{1}{2}(1 - y) - \sqrt{4 - y} \right)\, dy$

(e) $\int_{-3}^{1} (3 - 2x + x^2)\, dx$

14. The volume of the solid obtained by rotating the region bounded by the graphs of $y^2 = x$ and $y = x - 2$, about the y-axis is given by the definite integral

(a) $\pi \int_{-1}^{2} (4 + 4y + y^2 - y^4)\, dy$

(b) $\pi \int_{1}^{4} (x^2 - 3x + 4)\, dx$

(c) $\pi \int_{-1}^{2} (4 - 4y - y^2 - y^4)\, dy$

(d) $\pi \int_{1}^{4} (x^2 + 3x - 4)\, dx$

(e) $\pi \int_{-2}^{1} (4 - 4y + y^2 - y^4)\, dy$

15. The volume of the solid obtained by rotating the region bounded by the graphs of $y = -\frac{1}{x}$, $y = \frac{1}{x}$, $x = 1$, and $x = 2$, about $y = 1$, is equal to

(a) $4\pi \ln 2$

(b) $4\pi \ln 2 + 1$

(c) $\pi \ln 2$

(d) $\pi \ln 2 + 4$

(e) $8\pi \ln 2$

16. The value of $\int_1^2 x\left(e^{x^2-1} - \frac{1}{2x}\right)\,dx$ is equal to

(a) $\frac{1}{2}e^3 - 1$

(b) $3e^3 - 1$

(c) $\frac{1}{2}e^3 - e - 1$

(d) $\frac{1}{2}e^3 - e^2 + e - 1$

(e) $\frac{1}{2}e^3 - \ln 2$

17. Which one of the following inequalities is **TRUE**?

(a) $\int_{\pi/16}^{\pi/8} \cos^2 x \, dx \geq \int_{\pi/16}^{\pi/8} \cos^3 x \, dx$

(b) $\int_{\pi/16}^{\pi/8} \sin^3 x \, dx \geq \int_{\pi/16}^{\pi/8} \sin^2 x \, dx$

(c) $\int_{\pi/16}^{\pi/8} \tan x \, dx \geq \int_{\pi/16}^{\pi/8} \cot x \, dx$

(d) $\int_{\pi/16}^{\pi/8} \sin x \, dx \geq \int_{\pi/16}^{\pi/8} \cos x \, dx$

(e) $\int_{\pi/16}^{\pi/8} (\sin x - 1) \, dx \geq 0$

18. The value of $\int_{-2}^{2} (x^3 + 1)\sqrt{4 - x^2} \, dx$ is equal to

[Hint: One term of the integral may be interpreted as an area].

(a) 2π

(b) 4π

(c) $3\pi - 2$

(d) $\pi + 2$

(e) $\frac{1}{2}\pi - 4$

19. The area enclosed by the graphs of $y = \sin x$, $y = \sin 2x$, $x = 0$ and $x = \dfrac{\pi}{2}$ is equal to

(a) $\dfrac{1}{2}$

(b) 1

(c) $\dfrac{3}{2}$

(d) $\dfrac{3}{4}$

(e) $\dfrac{3}{8}$

20. Let S be a solid whose base is enclosed by the graphs of $y = x^2$ and $y = 1$ and whose cross-sections perpendicular to the y-axis are squares. Then the volume of S is

(a) 2

(b) 4

(c) $\dfrac{3}{2}$

(d) $2\sqrt{2}$

(e) 8