

1. A particle moves along the curve $xy + y^2 = 35$. As it reaches the point $(2, 5)$ the x -coordinate is increasing at a rate of 2 cm/sec . How fast is the y -coordinate changing at that instant?

a) $-\frac{5}{6} \text{ cm/sec}$.

b) -3 cm/sec .

c) $-\frac{7}{6} \text{ cm/sec}$.

d) 3 cm/sec .

e) $\frac{1}{6} \text{ cm/sec}$.

2. The slope of the tangent line to the graph of $y = (1 + 4x^2)\tan^{-1}(2x)$ at $x = \frac{1}{2}$ is

a) $2 + \pi$

b) $4 + \pi$

c) $8 + 2\pi$

d) $2 - \pi$

e) $1 + 2\pi$

3. If $y = (\sqrt{x} - \frac{1}{\sqrt{x}})^2$, then $y'' =$

- a) $2x^{-3}$
- b) $2\sqrt{x^3}$
- c) $-3x^{-2}$
- d) $-2\sqrt{x^3}$
- e) $4x^{-3}$

4. If $y = \frac{\tan x}{1 + x \tan x}$, then $y' = \frac{g(x)}{(1 + x \tan x)^2}$ where $g(x) =$

- a) 1
- b) $\sec x \tan x$
- c) $\sec x$
- d) $\tan x$
- e) -2

5. The limit $\lim_{x \rightarrow 2} \frac{\tan(x-2)}{4x^2 - 8x}$

a) is equal to $\frac{1}{8}$

b) is ∞

c) is $-\infty$

d) is equal to $\frac{1}{4}$

e) is equal to 1

6. If $y = \sqrt{\sin(\csc \pi x)}$, then the product $2yy'$ is equal to

a) $-\pi \cos(\csc \pi x) \cdot \csc \pi x \cdot \cot \pi x$

b) π

c) $-\pi$

d) $2\sin \pi x \cdot \cos \pi x$

e) $\cos(\csc \pi x \cdot \cot \pi x)$

7. If $f'(x) = \lim_{h \rightarrow 0} \frac{e^{2x}(e^{2h} - 1)}{3h}$, then a possible expression for the function f is

a) $f(x) = \frac{1}{3}e^{2x}$

b) $f(x) = \frac{1}{3}(e^x + 1)^2$

c) $f(x) = 3e^{2x}$

d) $f(x) = 3(e^{2x} - 1)$

e) $f(x) = e^{2x} + \frac{1}{3}$

8. If $(0, \alpha)$ are the coordinates of the y -intercept of the tangent line to the graph of $y = \sin^{-1}(2x)$ at $x = \frac{\sqrt{2}}{4}$, then $\alpha =$

a) $\frac{\pi}{4} - 1$

b) $\frac{\pi}{4} - \sqrt{2}$

c) $\frac{\pi}{4} - \frac{1}{4}$

d) $\frac{\pi}{4} + \sqrt{2}$

e) $\pi\sqrt{2}$

9. If a particle is moving according to a law of motion $S(t) = 3 \cos\left(\frac{\pi}{2}t\right)$ where t is measured in seconds and S in meters, then the total distance traveled by the particle during the time interval $[0, 5]$ is

- a) 15 meters
- b) 6 meters
- c) 12 meters
- d) 18 meters
- e) 9 meters

10. If $y = \ln\left[\frac{2}{3}\left(x + \sqrt{x^2 - 1}\right)\right]$, then $\frac{dy}{dx} =$

- a) $\frac{1}{\sqrt{x^2 - 1}}$
- b) $\frac{2}{3\sqrt{x^2 - 1}}$
- c) $\frac{3}{2\sqrt{x^2 - 1}}$
- d) $\frac{2}{3}\left(1 + x\sqrt{x^2 - 1}\right)$
- e) $\frac{1}{x + \sqrt{x^2 - 1}}$

11. The volume of a sphere is changing at the rate of $4\pi \text{ cm}^3/\text{sec}$. How fast is the diameter of the sphere changing when its volume is $\frac{32\pi}{3} \text{ cm}^3$?

[Volume of a sphere = $\frac{4\pi}{3}(\text{radius})^3$].

- a) $\frac{1}{2} \text{ cm}/\text{sec}$.
 - b) $\frac{3}{2} \text{ cm}/\text{sec}$.
 - c) $\frac{3}{8} \text{ cm}/\text{sec}$.
 - d) $\frac{3}{4} \text{ cm}/\text{sec}$.
 - e) $\frac{9}{8} \text{ cm}/\text{sec}$.
12. The slope of the normal line to the curve $e^{\frac{y}{x}} = x - y$ at the point $(1, 0)$ is

- a) -2
- b) 1
- c) -1
- d) $\frac{1}{3}$
- e) -3

13. If $f(x) = x^{\ln x}$, then $f'(e^2) =$

- a) $4e^2$
- b) $2e^2$
- c) e^2
- d) $\frac{1}{2}e^2$
- e) $\frac{1}{4}e^2$

14. If $f(x) = 2^x \cdot x^2$, then $f'(2) =$

- a) $16(1 + \ln 2)$
- b) $8(2 + \ln 2)$
- c) $16(\ln 2 - 1)$
- d) $\ln 16 + \ln 2$
- e) $4(1 + \ln 2)$

15. $\lim_{x \rightarrow 0} \frac{5x^2}{2x - 2x \cos x + 2 \sin^2 3x}$

- a) is equal to $\frac{5}{18}$
- b) does not exist
- c) is equal to 0
- d) is equal to $\frac{5}{3}$
- e) is equal to 5

16. If $f(x) = xe^{-x}$, then $f^{(100)}(x) = (Ax + B)e^{-x}$, where A and B are constants and $A + B =$

- a) -99
- b) 99
- c) -2
- d) 101
- e) -100

17. If the tangent line to the graph of $y = x^2 - 4x + 19$ at the point $P(\alpha, \beta)$, where $\alpha < 0$, passes through the point $(3, 0)$, then $\beta =$

- a) 24
- b) 18
- c) 30
- d) 28
- e) 36

18. Which one of the following statements is TRUE about the function $f(x) = \sqrt[3]{x}$?

- a) The graph of f has vertical tangent at $x = 0$
- b) $\lim_{x \rightarrow 0^-} f'(x) = -\infty$
- c) The graph of f has a vertical asymptote at $x = 0$
- d) f is differentiable at $x = 0$
- e) The graph of f' lies below the x -axis on $(-\infty, 0)$

19. Given the function $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ |x + 1| & \text{if } -2 \leq x \leq 8. \\ (x - 6)^2 & \text{if } x > 8 \end{cases}$

The sum of all values of x for which the function f is not differentiable is

- a) 5
- b) 6
- c) 0
- d) 3
- e) 4

20. Which one of the following statements is TRUE?

- a) If f' is differentiable at a , then $f''(a)$ exists
- (b) If $\lim_{x \rightarrow a} f'(x)$ exists, then f' is continuous at a
- (c) If f is continuous at a , then f is differentiable at a
- (d) If $f(a) = 0$ and $f'(a) = 0$, then $f''(a) = 0$.
- (e) If $f'(a)$ does not exist, then f is discontinuous at a .