

King Fahd University of Petroleum and Minerals

Department of Mathematics

Math 102 Sem II (052)

Final Exam **B**

Sun 4 / 6 / 2006

Time $2\frac{1}{2}$ hours

Name: _____ I.D.#: _____ Serial #: _____

Section #

17

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Answer all the questions

For the solving part show all of your work

For the multiple choice part mark your answer in the following table

1. If the region bounded by the curves $y = x^2$, x -axis, and $x = 1$ is revolving about $x = -2$, then the volume is expressed by the integral

a. $V = 2\pi \int_0^2 (x-1)x^2 dx$

b. $V = 2\pi \int_0^1 (x^2 - 2)x dx$

c. $V = 2\pi \int_0^2 (x+1)x^2 dx$

d. $V = 2\pi \int_0^1 (x+2)x^2 dx$

e. $V = 2\pi \int_0^2 (x^2 - 1)x dx$

2. The arclength of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ for $x = 0$ to $x = 3$ is equal to :

a. $\frac{14}{3}$

b. $\frac{26}{7}$

c. $\frac{124}{15}$

d. $\frac{15}{4}$

e. $\frac{22}{5}$

3. One of the following is TRUE

a. $\int \frac{2}{x^2 - 1} dx = \int \frac{dx}{x-1} - \int \frac{dx}{x+1}$

b. The series $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \frac{1}{1 - \frac{3}{2}}$

c. $\int_0^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$

d. The integral $\int_1^{\infty} \frac{1}{x} dx$ is convergent.

e. The sequence $\left\{ \frac{\sin n}{n} \right\}_{n=1}^{\infty}$ is divergent

4. The area of the region bounded by $y = \ln x$, x -axis, and $x = e$ is :

Notation

a. e

b. $\frac{1}{2}$

c. $\ln(e-1)$

d. $e-1$

e. 1

5. $\int_{\frac{6}{\pi}}^{\frac{3}{\pi}} \frac{1}{x^2 \sin \frac{1}{x}} dx$

a. $2 - \frac{2}{\sqrt{3}}$

b. $\frac{1}{2} - \frac{2}{\sqrt{3}}$

c. $\frac{\sqrt{3}}{2} - \ln 2$

d. $\ln \sqrt{3} + 2$

e. $\ln \sqrt{3} + \ln(2 - \sqrt{3})$

6. $\int_0^2 \frac{dx}{(x-1)^2} =$

a. 2

b. $\frac{1}{2}$

c. Divergent

d. -1

e. -2

7. $\int_0^{\frac{\pi}{3}} \tan^3 x \sec^4 x \, dx =$

a. $\frac{9}{2}$

b. $\frac{9\sqrt{3}}{2}$

c. $\frac{27}{2}$

d. $\frac{27}{4}$

e. $\frac{9}{4}$

8. $\frac{d}{dx} \int_{\sqrt{x}}^{\frac{1}{x}} \frac{\tan t}{t^2 + 1} \, dt$

a. $\ln \frac{1}{x} - \ln \sqrt{x}$

b. $\frac{-\tan \frac{1}{x}}{1 + x^2} - \frac{\tan \sqrt{x}}{2x\sqrt{x} + 2\sqrt{x}}$

c. $\frac{-1}{(1 + x^2)^2}$

d. $\tan^{-1} \frac{1}{x} - \tan^{-1} \sqrt{x}$

e. $\frac{\tan^2 \frac{1}{x}}{2} - \frac{\tan^2 \sqrt{x}}{2}$

9. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n n^2}{10^n}$ is equal to:

- a. 5
- b. 10
- c. $\sqrt{10}$
- d. 1
- e. 2

10. $\int_0^1 \sin^{-1} x \, dx =$

- a. $\frac{\pi}{2}$
- b. $\frac{\pi}{2} - 1$
- c. $\frac{\pi}{4}$
- d. $\frac{\pi}{4} + 1$
- e. 1

11. $\int_{-\frac{\pi}{2}}^0 \frac{\cos \theta}{\sin^2 \theta + 2 \sin \theta + 2} d\theta$

- a. $\ln 5 - 1$
- b. $\ln 5 - \ln 2$
- c. $\frac{\pi}{4}$
- d. $\frac{\pi}{2}$
- e. 1

12. $\int_0^{\ln 2} \frac{dx}{\sqrt{e^{2x} - 1}} =$

- a. $\frac{\sqrt{e^4 - 1}}{2}$
- b. $\frac{\pi}{4}$
- c. 1
- d. $2\sqrt{e^4 - 1}$
- e. $\frac{\pi}{3}$

13. Let the series $\sum_{k=1}^{\infty} \left[\frac{1}{2k-1} - \frac{1}{2k+1} \right]$ then the n th partial sum of the series S_n is equal to:

a. $S_n = \frac{2}{3} - \frac{1}{2n+1}$

b. $S_n = \frac{3}{4} - \frac{1}{2n+1}$

c. $S_n = 1 - \frac{1}{2n+1}$

d. $S_n = \frac{3}{2} - \frac{1}{2n+1}$

e. $S_n = \frac{1}{2} - \frac{1}{2n+1}$

14. If the parametric curve $x = \cos^2 t$, $y = \sin^2 t$, $0 \leq t \leq \frac{\pi}{2}$, is revolving about the x -axis, then the surface area is equal to :

a. $\frac{\pi}{2}$

b. $\frac{\pi}{\sqrt{2}}$

c. π

d. $\pi\sqrt{2}$

e. 2π

Solving Part (Show all of your work)

15. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$

16. $\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx =$

17. $\int_0^{\ln 2} \frac{\tanh x}{1 + \sinh^2 x} dx =$

18. Find the volume of revolution if the region between the curve $y = \cos x$, and x -axis from $x = 0$ to $x = \frac{\pi}{2}$ is revolving about the x -axis

19. Determine if the series $\sum_{n=0}^{\infty} \frac{3^n + 2^n}{7(5)^n}$ is convergent or divergent, give the sum if convergent.

20. Use the fact that $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, for $|x| < 1$, find the power series representation of $\tan^{-1}(2x)$, $|x| < \frac{1}{2}$

21. Determine if the sequence $\left\{ \frac{n!}{n^n} \right\}_{n=1}^{\infty}$ is convergent or divergent