

King Fahd University of Petroleum and Minerals  
Department of Mathematical Sciences  
**Final Exam Math 102 (Sections 11 and 15)**  
Semester II, 2005–2006 (052)

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Name: \_\_\_\_\_

ID #: \_\_\_\_\_ Sec. #: \_\_\_\_\_

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1. Prove that the harmonic series  $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}$  diverges. (2 points)

2. Determine whether the sequence  $\left\{ \frac{n}{2^n} \right\}_{n=1}^{+\infty}$  converges or diverges. If it converges, find its limit. (2 points)

3. Find the first four nonzero terms of the Binomial series for  $\sqrt{1+x^4}$ . (2 points)

4. Determine whether the alternating series  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{\sqrt{k+1}}$  converges or diverges. (2 points)

5. Evaluate  $\int \frac{dx}{\sqrt{x-1} + (x-1)^{3/2}}$ . (2 points)

Please write your answer in the box below for questions 6 to 15.

6	7	8	9	10	11	12	13	14	15

6.  $\int_{-1}^2 \sqrt{2 - |x|} dx$  is equal to (3 points)

(a)  $\frac{2}{\sqrt{3}}$

(b)  $\frac{4\sqrt{2}}{3}$

(c)  $\frac{-2}{3}$

(d)  $\frac{8\sqrt{2} - 2}{3}$

(e)  $\frac{8\sqrt{2}}{3}$

7. If the region bounded by  $y = 3 - 2x$ ,  $y = 2$  and  $x = 0$  is revolved about the  $y$ -axis, then the volume of the solid is equal to (3 points)

(a)  $\frac{13\pi}{6}$

(b)  $\frac{\pi}{12}$

(c)  $\frac{37\pi}{12}$

(d)  $\frac{13\pi}{12}$

(e)  $\frac{31\pi}{12}$

8.  $\int_{\ln 2}^{\ln 4} \frac{dx}{\sqrt{e^x - 1}}$  is equal to (3 points)

(a)  $\tan^{-1} \sqrt{3}$

(b)  $2(\tan^{-1} \sqrt{3} + 5)$

(c)  $\frac{\pi}{4}$

(d)  $2 \tan^{-1} \sqrt{5}$

(e)  $\frac{\pi}{6}$

9. The interval of convergence of the power series  $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} x^k$  is (3 points)

(a)  $[-1, 1)$

(b)  $(-1, 1)$

(c)  $(-1, 1]$

(d)  $[-1, 1]$

(e)  $(-\infty, \infty)$

10. The exact arc length of the parametric curve  $x = (1+t)^2$ ,  $y = (1+t)^3$ ,  $0 \leq t \leq 1$  is equal to (3 points)

(a)  $\frac{1}{27}(\sqrt{10} - \sqrt{13})$

(b)  $\frac{\pi}{27}$

(c)  $\frac{1}{27}(80\sqrt{10} - 13\sqrt{13})$

(d)  $40\sqrt{10} - 13$

(e)  $8\sqrt{15} + 13\sqrt{13}$

11.  $\int_{-2}^7 \frac{dx}{(x+1)^{2/3}}$  is equal to (3 points)

(a)  $\frac{3}{4}$

(b) 9

(c)  $\frac{19}{2}$

(d) 10

(e) 6

12. The series  $\sum_{k=1}^{\infty} \frac{1}{1+k^2}$  converges to (3 points)

(a)  $\ln 3$

(b)  $\frac{3\pi}{4}$

(c)  $\frac{\pi}{2}$

(d)  $\frac{3\pi}{2}$

(e)  $\frac{\pi}{4}$

13. The area of the region enclosed by the curve  $y = \sqrt{2-x}$  and  $y = -x$  is equal to (3 points)

(a)  $\sqrt{3}$

(b)  $\sqrt{10}$

(c)  $\frac{3\pi}{2}$

(d)  $\frac{10}{3}$

(e)  $\ln 3$

14.  $\int \frac{(5x - 5)dx}{3x^2 - 8x - 3}$  is equal to (3 points)

(a)  $\ln|x - 3| + \frac{2}{3}\ln|3x + 1| + c$

(b)  $\frac{2}{3}\ln|3x - 1| + c$

(c)  $\ln|x - 3| + c$

(d)  $\ln|3x + 1| + 5x + c$

(e)  $\ln|x + 3| + \frac{2}{3}\ln|3x + 1| + c$

15.  $\int_0^7 \frac{x dx}{(x + 1)^{1/3}}$  is equal to (3 points)

(a)  $\frac{75}{10}$

(b)  $\frac{78}{5}$

(c)  $\frac{141}{10}$

(d) 3

(e)  $\frac{96}{5}$