King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematical Sciences

FINAL EXAM

(MATH. 102-043 Sections 1 & 2)

Name:
ID:

Prob. 1

Calculate $\int_0^{\ln\sqrt{2}} \frac{1+\cos(\frac{1}{e^{2x}})}{e^{2x}} dx$

Prob. 2

Evaluate the following limit by interpreting it as a Riemann sum in which the given interval is divided into n subintervals of equal width

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{\pi}{4n}\sec^2\left(\frac{\pi k}{4n}\right);\ [0,\pi/4].$$

Prob. 3

(a) Give a geometric argument to show that

$$\frac{1}{x+1} < \int_{x}^{x+1} \frac{dt}{t} < \frac{1}{x}, \; x > 0$$

(b) Use the result in part (a) to prove that

$$\frac{1}{x+1} < \ln\left(1+\frac{1}{x}\right) < \frac{1}{x}, \; x > 0$$

(c) Use the result in part (b) to prove that

$$e^{\frac{x}{x+1}} < \left(1+\frac{1}{x}\right)^{x} < e$$

and hence that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$.

(d) Use the inequality in part (c) to prove that

$$\left(1+\frac{1}{x}\right)^{x} < e < \left(1+\frac{1}{x}\right)^{x+1}, \; x > 0.$$

Prob. 4

Find the area of the region enclosed between the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the coordinate axes.

Prob. 5

Use cylindrical shells to find the volume obtained by revolving the area in the first quadrant between $x = 3y^3 - 4y^4$ about the x-axis

Prob. 6

Find by two different methods the volume when the region bounded by $y = \frac{1}{3}\cos x$ and $y = 3x^4$ is revolved about the

a) $x = 4$

b) $y = 3$

c) x-axis

d) y-axis

Prob. 7

What is the nature of

(a) $\sum\limits_{k=1}^{\infty}\left(\frac{k!}{k^k}\right)^k$, (b) $\sum\limits_{k=2}^{\infty}\frac{k}{(\ln k)^k}$

Prob. 8

What is the nature of

(a) $\sum_{k=1}^{\infty} \frac{(2k!)^2 2^k}{(5k+5)!}$, (b) $\sum_{k=1}^{\infty} \frac{\sqrt{k}\ln\sqrt{k}}{k^3+1}$

Prob. 9

Classify the series as absolutely convergent, conditionally convergent or divergent

(a) $\sum\limits_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{k^3+2}$, (b) $\sum\limits_{k=1}^{\infty} \sin \frac{2k\pi}{3}$, (c) $\sum\limits_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k+1}+\sqrt{k}}$, (d) $\sum\limits_{k=1}^{\infty} \frac{(-1)^{k+1}(k+7)!}{(5k-1)!}$