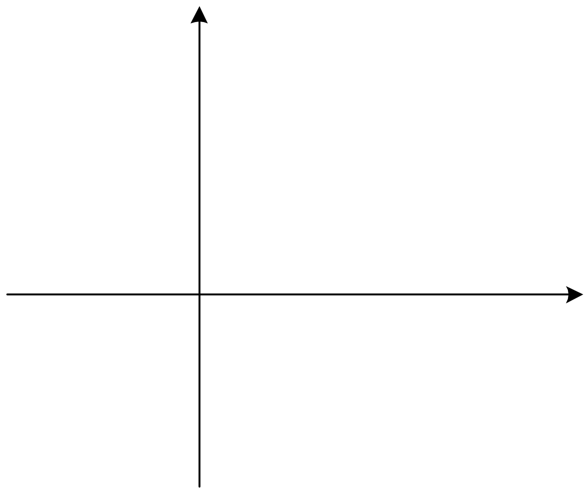
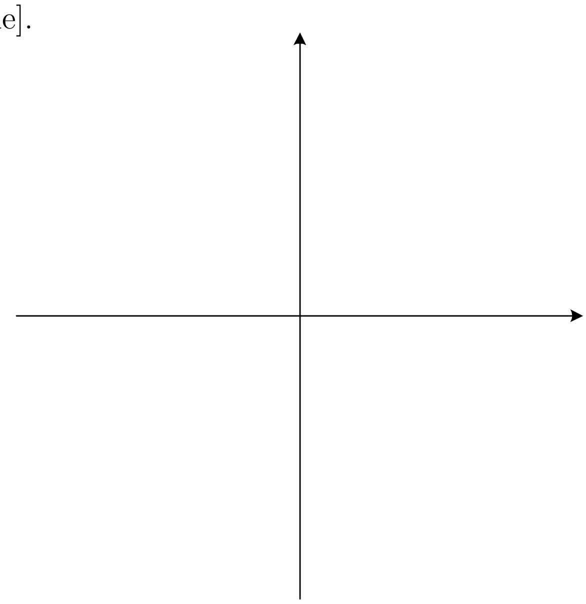
1. Using **the method of cylindrical shells**, set up, but do not evaluate, an integral for the volume of the solid obtained by rotating

   (a) [**7 points**] the region bounded by the curves $y = \sqrt{x}$, $y = x - 2$ and $y = 0$ about the x-axis. [Sketch the region and a typical rectangle].



   (b) [**7 points**] the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = -1$. [Sketch the region and a typical rectangle].



2. [**5 points**] Find the average value of the function $f(t) = \tan t \sec\ t$ over the interval $\left[0, \frac{\pi}{4}\right]$.

3. Determine whether the integral is convergent or divergent. If it is convergent, find its value.

   (a) [**6 points**] $\displaystyle\int_0^9 \frac{1}{x\sqrt{x}}\,dx\,.$

   (b) [**8 points**] $\displaystyle\int_0^{+\infty} x\,e^{-10x}\,dx\,.$

4. [**6 points**] Determine whether the sequence $\left\{ \dfrac{(-1)^n \sqrt{n}}{n+7} \right\}_{n=1}^{+\infty}$ is convergent or divergent. If it is convergent, find its limit.

5. [**7 points**] Use geometric series to write the number

$$1.2\overline{13} = 1.2131313\ldots$$

as a ratio of two integers.

6. Evaluate the following integrals:

   (a) [**9 points**] $\int x(\ln x)^2 \, dx.$

   (b) [**10 points**] $\int \frac{x^3}{\sqrt{4-x^2}} \, dx.$

(c) [**12 points**] $\displaystyle\int \frac{x^3+1}{x^3+x}\,dx.$

(d) [**10 points**] $\displaystyle\int \frac{\sec x}{2+\tan x}\,dx.$ Hint: Use the substitution $t = \tan\left(\dfrac{x}{2}\right)$.

7. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

   (a) [**6 points**] $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{1}{2}\right)^{\frac{1}{n^2}}$.

   (b) [**7 points**] $\displaystyle\sum_{n=1}^{+\infty} [\tan^{-1}(2n-1) - \tan^{-1}(2n+1)]$.