King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 001 | **Math 102** | CODE 001

# FINAL EXAM
## Term 083
## Thursday 3/9/2009
## Net Time Allowed: 180 minutes

Name: ______________________________

ID: ____________________ Sec: ____________________.

**Check that this exam has 28 questions.**

## Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The volume of the solid generated by revolving the region bounded by the curves

$$y = x^3, x = 0, \text{ and } y = 1$$

about the line $y = 2$ is given by

(a) $V = \int_0^1 \pi \cdot [(2 - x^3)^2 - 2^2]dx$

(b) $V = \int_0^1 \pi[(\sqrt[3]{y})^2 - 1]dy$

(c) $V = \int_0^1 \pi \cdot (x^6 - 1)dx$

(d) $V = \int_0^1 \pi \cdot [(2 - x^3)^2 - 1]dx$

(e) $V = \int_0^1 \pi[(2 - \sqrt[3]{y})^2 - 1]dy$

2. $\int \sin^2 x \cos^3 x \, dx =$

(a) $\frac{1}{12} \sin^3 x \cos^4 x + C$

(b) $\frac{1}{6} \sin^3 x - \frac{1}{10} \cos^5 x + C$

(c) $\frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$

(d) $\sin^2 x - \sin^4 x + C$

(e) $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$

3. The area of the surface obtained by rotating the curve

$$y = \ln x, \quad 1 \le x \le 3$$

about the $y$-axis is given by

(a) $\displaystyle\int_1^3 2\pi x\sqrt{x^2+1}\,dx$

(b) $\displaystyle\int_0^{\ln 3} 2\pi y\sqrt{1+e^{2y}}\,dy$

(c) $\displaystyle\int_1^3 2\pi \frac{\sqrt{x^2+1}}{x}\,dx$

(d) $\displaystyle\int_1^3 2\pi(\ln x)\sqrt{x^2+1}\,dx$

(e) $\displaystyle\int_1^3 2\pi\sqrt{x^2+1}\,dx$

4. The sequence $\left\{2 - \dfrac{\cos n}{2^n}\right\}_{n=1}^{+\infty}$

(a) converges to 3

(b) converges to 1

(c) diverges

(d) converges to $-1$

(e) converges to 2

5. The series $\sum_{n=1}^{+\infty} \frac{n^2-1}{n^2+1}$ is

(a) divergent by the ratio test

(b) convergent by the integral test

(c) divergent

(d) convergent

(e) convergent by comparing it with a suitable $p$-series

6. The area of the region enclosed by the curves

$$y = x^2 - 1 \text{ and } y = x + 1$$

is equal to

(a) $2$

(b) $\frac{3}{2}$

(c) $4$

(d) $\frac{9}{2}$

(e) $-3$

7. The first four terms of the Taylor series of $f(x) = \dfrac{1}{\sqrt{x}}$ about $a = 1$ are given by

(a) $1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{5}{16}(x-1)^3$

(b) $1 - (x-1) + (x-1)^2 + (x-1)^3$

(c) $1 + \frac{1}{2}(x-1) - \frac{3}{8}(x-1)^2 - \frac{2}{3}(x-1)^3$

(d) $1 - \frac{1}{2}(x-1) + \frac{3}{4}(x-1)^2 - \frac{15}{8}(x-1)^3$

(e) $\frac{1}{2} - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{15}{7}(x-1)^3$

8. If $F(x) = \displaystyle\int_1^{x^3} \tan^{-1}(\sqrt[3]{t})\, dt$, then $F(1) + F'(1) + F''(1) =$

(a) $\frac{3\pi}{2} - \frac{1}{2}$

(b) $\frac{9\pi}{4}$

(c) $\frac{9\pi}{4} + \frac{3}{2}$

(d) $\frac{3\pi}{2} + \frac{3}{2}$

(e) $\frac{7\pi}{4} + \frac{1}{2}$

9. $\displaystyle\int \frac{2x^2}{\sqrt[3]{1+x^3}}\,dx =$

(a) $2(1+x^3)^{1/3}+C$

(b) $(1+x^3)^{2/3}+C$

(c) $-\frac{1}{4}(1+x^3)^{-4/3}+C$

(d) $\frac{2}{3}(1+x^3)^{-2/3}+C$

(e) $2\ln|\sqrt[3]{1+x^3}|+C$

10. If $f(x) = \begin{cases} \sqrt{3-x} & \text{if } x \leq 2 \\ e^{x-2} & \text{if } x > 2, \end{cases}$

then $\displaystyle\int_{-1}^{3} f(x)dx =$

(a) $e+\frac{11}{3}$

(b) $\frac{1}{2}e+\frac{4}{3}$

(c) $e+\sqrt{2}$

(d) $2e-\frac{14}{3}$

(e) $e-8$

11. If $\dfrac{3x^2+2x+1}{(x-1)(x^2+2x+5)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+2x+5}$,
then $A+B-C =$

(a) $-\dfrac{3}{4}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{1}{3}$

(d) $-\dfrac{1}{2}$

(e) $0$

12. The sum of the series

$$-\frac{2^4}{4!} + \frac{2^6}{6!} - \frac{2^8}{8!} + \frac{2^{10}}{10!} - \cdots$$

is equal to

(a) $-2 - \sin 2$

(b) $1 - \cos 2$

(c) $-\dfrac{1}{2} - 2\cos 2$

(d) $2 - \cos 2$

(e) $-1 - \cos 2$

13. The volume of the solid obtained by rotating the region enclosed by the curves

$$y = \frac{x}{1+x^6}, \quad y = 0, x = 0, \text{ and } x = 1$$

about the $y$-axis is equal to

(a) $\frac{\pi^2}{12}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi^2}{6}$

(d) $\frac{2\pi}{3}$

(e) $\frac{2\pi^2}{3}$

14. $\int_0^{\pi/2} e^{\sin t} \sin(2t)dt =$

(a) 2

(b) $-\frac{1}{2}$

(c) 0

(d) $-3$

(e) 4

15. The area of the region bounded by the graph of $f(x) = 2^x - 2$ and the $x$-axis from $x = 0$ to $x = 2$ is equal to

(a) $\dfrac{5}{\ln 2}$

(b) $\dfrac{1}{\ln 2}$

(c) $\dfrac{2}{\ln 2}$

(d) $\dfrac{4}{\ln 2}$

(e) $\dfrac{3}{\ln 2}$

16. The area of the surface generated by revolving the curve

$$y = \sqrt{x}, \quad 0 \le x \le 2$$

about the $x$-axis is equal to

(a) $\dfrac{2\pi}{3}(3\sqrt{3} - 2)$

(b) $\dfrac{2\pi}{3}(3\sqrt{3} - 16)$

(c) $13\pi$

(d) $\dfrac{19\pi}{3}$

(e) $\dfrac{13\pi}{3}$

17. The length of the curve

$$y = \int_0^x \sqrt{9\sin^2 t - 1}\,dt, \quad 0 \le x \le \frac{\pi}{2}$$

is equal to

(a) $\frac{1}{3}$

(b) $\sqrt{3}$

(c) $3$

(d) $\frac{3}{2}$

(e) $9$

18. $\int \frac{\sec^2 t}{\tan^2 t + \tan t}\,dt =$

(a) $\ln|1 + \cot t| + C$

(b) $\ln|1 + \tan t| + C$

(c) $\ln|\tan^2 t + \tan t| + C$

(d) $\frac{1}{1 + \tan t} + C$

(e) $\ln\left|\frac{\tan t}{1 + \tan t}\right| + C$

19. If $f$ is an even function and $\int_0^2 f(x)\,dx = 3$,

then $\int_{-1}^{1} [xf(x) + f(2x)]dx =$

(a) $-6$

(b) $3$

(c) $6$

(d) $2$

(e) $\frac{3}{2}$

20. The interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

is given by

(a) $(-\infty, \infty)$

(b) $\left[\frac{5}{2}, \frac{7}{2}\right)$

(c) $\left[\frac{5}{2}, \frac{7}{2}\right]$

(d) $\left(\frac{5}{2}, \frac{7}{2}\right)$

(e) $(2, 4)$

21. The series $\sum_{n=1}^{+\infty} \frac{(n+1)!}{3^{n-1} \cdot [5 \cdot 7 \cdot 9 \cdots (2n+3)]}$ is

(a) divergent by the test of divergence

(b) convergent by the ratio test

(c) convergent by the test of divergence

(d) divergent by the ratio test

(e) a series with which the ratio test is inconclusive

22. The error in approximating the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n+2}}$ by the sum of the first 29 terms is less than or equal to

(a) $\frac{2}{5}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{5}$

(e) $\frac{1}{\sqrt[5]{33}}$

23. $\displaystyle\int \frac{e^{-2x}-1}{x}\,dx =$

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^n x^n}{n\cdot n!} + C$

(b) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^{n-1} x^{n-1}}{(n-1)!} + C$

(c) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^{n-1} x^{n-1}}{(n-1)!} + C$

(d) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 2^n x^{n+1}}{(n+1)!} + C$

(e) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^n x^n}{(n+1)!} + C$

24. For $|x| < 1$, $\displaystyle\sum_{n=2}^{+\infty} n(n-1)x^{n-2} =$

(a) $\dfrac{2}{(1-x)^3}$

(b) $\dfrac{2x}{(1-x)^3}$

(c) $\dfrac{x}{1-x}$

(d) $\dfrac{1}{(1-x)^2}$

(e) $\dfrac{3}{(1-x)^2}$

25. The series $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n+28}$ is

(a) divergent

(b) absolutely convergent

(c) conditionally convergent

(d) convergent by the ratio test

(e) neither convergent nor divergent

26. $\int_0^1 \frac{d}{dx}\left(\frac{e^x}{x^2+1}\right) dx + \frac{d}{dx}\int_0^1 \frac{e^x}{x^2+1}\, dx =$

(a) $e+1$

(b) $\frac{1}{2}\, e+3$

(c) can not be evaluated

(d) $\frac{e-2}{2}$

(e) $e-\frac{1}{2}$

27. $\displaystyle\int \frac{1}{x\sqrt{x-4}}\,dx =$

(a) $\ln|x| + \ln(\sqrt{x-4}) + C$

(b) $\displaystyle\frac{1}{2}\ln\left|\frac{\sqrt{x-4}-2}{\sqrt{x-4}+2}\right| + C$

(c) $\displaystyle\tan^{-1}\left(\frac{\sqrt{x-4}}{2}\right) + C$

(d) $\displaystyle\frac{\sqrt{x-4}}{x} + C$

(e) $\displaystyle 2\tan^{-1}\left(\frac{\sqrt{x-4}}{2}\right) + C$

28. The improper integral $\displaystyle\int_0^{+\infty} \frac{e^{-1/x}}{x^2}\,dx$ is

(a) convergent and its value is 0

(b) convergent and its value is 1

(c) convergent and its value is $e^{-1}$

(d) convergent and its value is $1-e$

(e) divergent

| Q | MM | V1 | V2 | V3 | V4 |
|---|---|---|---|---|---|
| 1 | a | d | e | e | a |
| 2 | a | e | c | a | a |
| 3 | a | e | b | c | a |
| 4 | a | e | e | a | b |
| 5 | a | c | b | e | e |
| 6 | a | d | a | b | c |
| 7 | a | a | c | b | a |
| 8 | a | c | a | e | c |
| 9 | a | b | b | d | d |
| 10 | a | a | a | b | d |
| 11 | a | b | e | c | d |
| 12 | a | e | e | b | a |
| 13 | a | c | a | c | b |
| 14 | a | a | c | e | d |
| 15 | a | b | b | b | d |
| 16 | a | e | b | a | b |
| 17 | a | c | d | e | c |
| 18 | a | e | c | d | b |
| 19 | a | b | a | e | b |
| 20 | a | b | c | e | e |
| 21 | a | b | b | c | a |
| 22 | a | c | a | a | a |
| 23 | a | a | d | a | a |
| 24 | a | a | c | e | a |
| 25 | a | c | d | c | b |
| 26 | a | d | a | d | c |
| 27 | a | c | a | d | a |
| 28 | a | b | b | a | c |