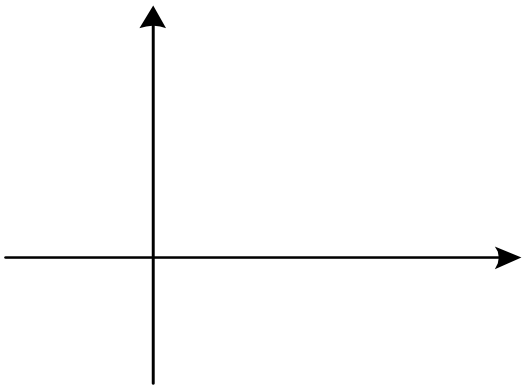
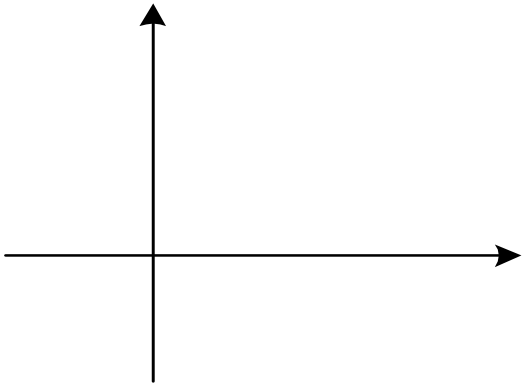
1. [**6 + 8 points**] Using **the method of cylindrical shells**, set up, but do do not evaluate, an integral for the volume of the solid obtained by rotating the region in the first quadrant bounded by the curves $y = x^2, y = 2 - x^2$, and $x = 0$

   (a) about the y-axis. [Sketch the region and a typical rectangle]



   (b) about the line $y = -2$. [Sketch the region and a typical rectangle.]



2. [**8 points**] Evaluate the integral or show that it is divergent:

$$\int_0^2 \frac{1}{(1-2x)^{4/3}}\, dx.$$

3. [**10 points**] Find the average value of the function $f(x) = (\sin^{-1} x)^2$ on the interval $[0, 1]$.

4. [**6 points**] Find a formula for the general term a_n of the sequence and determine whether the sequence converges or diverges:

$$\left\{ \frac{7}{2 \cdot 3}, \frac{7 \cdot 2^2}{3 \cdot 4}, \frac{7 \cdot 3^2}{4 \cdot 5}, \frac{7 \cdot 4^2}{5 \cdot 6} \cdots \right\}$$

5. Evaluate the following integrals.

(a) [**7 points**] $\displaystyle\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta.$

(b) [**10 points**] $\displaystyle\int \sqrt{5+4x-x^2}\,dx.$

(c) [**10 points**] $\displaystyle\int \frac{x^4+3x^3+2x^2+1}{(x^2+x)(x+2)}\,dx.$

(d) [**8 points**] $\displaystyle\int \frac{1}{1+\cos x-\sin x}\,dx.$

6. Determine whether the series is Convergent or Divergent. Justify.

(a) [**5 points**] $\displaystyle\sum_{n=0}^{+\infty} \frac{(\sqrt{2})^{3n}}{4^{n-1}}$.

(b) [**6 points**] $\displaystyle\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{n}\right)$.

(c) [**8 points**] $\displaystyle\sum_{n=2}^{+\infty} \frac{n}{n^2+2}$.

7. [**8 points**] Find the values of x for which the series $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{5x^2}{x^2+1}\right)^n$ converges. Then find the sum of the series for those values of x.