

King Fahd University of Petroleum and Minerals  
Department of Mathematics and Statistics

**Calculus II**  
**FINAL EXAM**  
**Semester II, Term 082**  
**Monday, June 22, 2009**

**EXAM COVER**

**Number of versions: 4**  
**Number of questions: 28**  
**Number of Answers: 5 per question**

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**FINAL EXAM**  
Semester II, Term 082  
Monday, June 22, 2009  
Net Time Allowed: 180 minutes

**MASTER VERSION**

1. The average value of the function  $f(x) = \cos^4 x \sin x$  over  $[0, \pi]$  is

(a)  $\frac{2}{5\pi}$

(b)  $\frac{2}{\pi}$

(c)  $\frac{2}{5}$

(d)  $\frac{5\pi}{2}$

(e)  $\frac{2}{3}$

2.  $\int \tan^4 x \, dx =$

(a)  $\frac{1}{3} \tan^3 x - \tan x + x + c$

(b)  $\frac{1}{3} \tan^3 x + \tan x \sec x + c$

(c)  $\ln |\sec x + \tan x| + \tan^3 x + c$

(d)  $\ln |\csc x - \sec x| + \tan x + c$

(e)  $\sec^3 x + 3 \sec^2 x + c$

3. The series  $\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$  is

- (a) divergent
- (b) convergent to 0.1
- (c) convergent to 100
- (d) convergent to 0.01
- (e) convergent to 0.001

4. For what values of  $p$ , is the series  $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^{p-4}}$  convergent?

- (a)  $p > 4$
- (b)  $p > 1$
- (c)  $p \geq 4$
- (d)  $p < 4$
- (e)  $p \leq 4$

5.  $\int \frac{1}{x^2 \sqrt{25 - x^2}} dx =$

(a)  $\frac{-1 \sqrt{25 - x^2}}{25 x} + c$

(b)  $\frac{1 \sqrt{25 - x^2}}{125 x} + c$

(c)  $\frac{-1 \sqrt{25 - x^2}}{5 x} + c$

(d)  $\frac{1 \sqrt{25 - x^2}}{5 x} + c$

(e)  $\frac{\sqrt{25 - x^2}}{x} + c$

6. The radius of convergence of the power series  $\sum_{n=0}^{+\infty} \frac{(x - 1)^n}{3^n}$  is

(a)  $R = 3$

(b)  $R = 1$

(c)  $R = \frac{1}{3}$

(d)  $R = \infty$

(e)  $R = 0$

7.  $\int \frac{x^2 - x + 6}{x^3 + x} dx =$

(a)  $6 \ln |x| - \frac{5}{2} \ln(x^2 + 1) - \tan^{-1}(x) + c$

(b)  $6 \ln |x| - \frac{5}{2} \ln(x^2 + 1) + \tan^{-1}(x) + c$

(c)  $6 \ln |x| - \frac{5}{2} \ln(x^2 + 1) - \sin^{-1}(x) + c$

(d)  $\ln |x| - \frac{1}{2} \ln(x^2 + 1) - \tan^{-1}(x) + c$

(e)  $-\frac{5}{2} \ln(x^2 + 1) - \tan^{-1}(x) + c$

8.  $\int_1^3 \frac{\sqrt{x}}{x^2 + x} dx =$

(a)  $\frac{\pi}{6}$

(b)  $\frac{\pi}{3}$

(c)  $2\pi$

(d)  $5\pi$

(e)  $\frac{5\pi}{4}$

9. The area of the region enclosed by the graphs of  $y = x - 1$  and  $x = (y - 1)^2$  is equal to

(a)  $\frac{9}{2}$

(b) 9

(c) 3

(d)  $\frac{8}{3}$

(e) 8

10. The improper integral  $\int_0^{\pi/2} \frac{\cos x}{1 - \sin x} dx$

(a) diverges

(b) converges and has the value 0

(c) converges and has the value  $\frac{\pi}{4}$

(d) converges and has the value  $\pi$

(e) converges and has the value  $\frac{\pi}{2}$

11. The area of the surface obtained by rotating the curve  $y = x^3$ ,  $0 \leq x \leq 1$  about the  $x$ -axis is

(a)  $\frac{\pi}{27}(10\sqrt{10} - 1)$

(b)  $\frac{\pi}{27}(145\sqrt{145} - 1)$

(c)  $\frac{\pi}{18}(10\sqrt{10} - 1)$

(d)  $\frac{\pi}{18}(145\sqrt{145} - 1)$

(e)  $\frac{\pi}{27}$

12. The sum of the series  $\sum_{n=1}^{\infty} \left[ \frac{3}{n(n+1)} + \frac{1}{2^n} \right]$  is equal to

(a) 4

(b) 3

(c) 2

(d) 1

(e) 5



13. The volume of the solid obtained by rotating the region enclosed by the curves  $y = \cosh x$ ,  $y = \sinh x$ ,  $x = 0$  and  $x = 5$ , about the  $x$ -axis, is (Hint:  $\cosh x - \sinh x > 0$ )

(a)  $5\pi$

(b)  $\frac{\pi}{5}$

(c)  $\frac{5\pi}{2}$

(d)  $\pi$

(e)  $\frac{\pi}{5} - 1$

14. Which one of the following statements is TRUE about the alternating series  $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n$ , where  $a_n = \frac{2n}{3n+1}$  ?

(a) The series is divergent

(b) The series is absolutely convergent

(c) The series is conditionally convergent

(d)  $a_{n+1} \leq a_n$  for all  $n$

(e)  $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n = \frac{2}{3}$

15. How many terms of the series  $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^3}$  do we need to add in order to ensure that the sum is accurate to within 0.001? (minimum number of terms)
- (a) 9
  - (b) 10
  - (c) 100
  - (d) 99
  - (e) 1000
16. By applying the Ratio Test to the series  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$ , we conclude that the
- (a) test fails
  - (b) series is convergent
  - (c) series is absolutely convergent
  - (d) series is conditionally convergent
  - (e) series is divergent

17. The interval of convergence of the power series  $\sum_{n=1}^{+\infty} \frac{(2x-3)^n}{n4^{2n}}$  is

(a)  $\left[-\frac{13}{2}, \frac{19}{2}\right)$

(b)  $\left(-\frac{13}{2}, \frac{19}{2}\right)$

(c)  $\left(-\frac{13}{2}, \frac{19}{2}\right]$

(d)  $\left[-\frac{13}{2}, \frac{19}{2}\right]$

(e)  $\left[\frac{13}{2}, \frac{19}{2}\right)$

18. By applying the Root Test to the series  $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{3^{1+3n}}$ , we conclude that the

(a) series is divergent

(b) test fails

(c) series is convergent

(d) series is absolutely convergent

(e) series is conditionally convergent

19. The Taylor series of  $f(x) = \frac{1}{x}$  about  $x = 2$  is

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x-2)^n$

(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^{n+1}$

(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x+2)^n$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x+2)^n$

20. The coefficient of  $x^4$  in the Maclaurin series of  $\cos^2 x$  is

(a)  $\frac{1}{3}$

(b)  $\frac{2}{3}$

(c) 2

(d)  $\frac{1}{2}$

(e)  $\frac{1}{4}$

21. Let us consider the series  $\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln^2 n)}$ . Then the integral test
- (a) implies that the series converges
  - (b) is not applicable because of the continuity condition
  - (c) implies that the series diverges
  - (d) is not applicable because of the decreasing condition
  - (e) is not applicable because of the positivity condition
22. Let us consider the sequence  $\{\tan^{-1}(-3n)\}$ . Then the sequence
- (a) converges and its limit is  $-\frac{\pi}{2}$
  - (b) converges and its limit is 0
  - (c) converges and its limit is  $\frac{\pi}{2}$
  - (d) diverges
  - (e) converges and its limit is  $-1$

23. The sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1} (2n+1)!}$  is

(a)  $\frac{1}{\sqrt{2}}$

(b)  $\sqrt{2}$

(c)  $\frac{1}{2}$

(d)  $\frac{\sqrt{3}}{2}$

(e)  $\sqrt{3}$

24.  $\lim_{h \rightarrow 0} \frac{1}{h} \int_2^{2+h} \sqrt{1+t^3} dt$

(a) is equal to 3

(b) is equal to 4

(c) is equal to  $-3$

(d) is equal to 0

(e) does not exist

25. The volume of the solid obtained by rotating the region enclosed by  $y = \frac{1}{x^2 + 2x + 2}$ ,  $x = 1$ ,  $x = 2$ , about the line  $x = -1$  is

(a)  $\pi \ln 2$

(b)  $2\pi \ln 2$

(c)  $\pi \ln 2 - 2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(d)  $2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(e)  $\pi(\tan^{-1} 2 - \tan^{-1} 3)$

26.  $\int_0^{\pi^2/4} \cos \sqrt{x} \, dx =$

(a)  $\pi - 2$

(b)  $\frac{\pi}{2} - 1$

(c)  $\frac{\pi}{2} - \frac{1}{2}$

(d)  $\frac{\pi^2}{4} - 1$

(e)  $\frac{\pi}{4} - 1$

27. The arc length of the curve  $y = x^2 - \frac{1}{8} \ln x$ ,  $1 \leq x \leq 3$  is equal to

(a)  $8 + \frac{1}{8} \ln 3$

(b)  $3 + \frac{1}{3} \ln 8$

(c)  $8 + \ln 3$

(d)  $3 + \ln 8$

(e)  $-8 + \frac{1}{8} \ln 3$

28. The power series representation for the function  $f(x) = \frac{x}{(x-2)^2}$  is  
(Hint: You may use differentiation)

(a)  $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} x^{n+1}$

(b)  $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} x^n$

(c)  $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+1}} x^{n+1}$

(d)  $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$

(e)  $\sum_{n=0}^{\infty} \frac{n+1}{2} x^{n+1}$