King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 001 | **Math 102** | **CODE 001**

Math 102
Final Exam
Term 081
Sunday, February 8, 2009
Net Time Allowed: 180 minutes

Name: ____

ID: ________________________ Sec: ________________________.

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Which one of the following is **TRUE**?

(a) $\sum_{n=1}^{+\infty} \frac{1}{n^{e-2}}$ is convergent

(b) $\sum_{n=1}^{+\infty} \frac{1}{n^{0.999}}$ is convergent

(c) $\sum_{n=1}^{+\infty} \frac{1}{n^{\pi/4}}$ is divergent

(d) $\sum_{n=1}^{+\infty} \frac{1}{n^{\sqrt{2}}}$ is divergent

(e) $\sum_{n=1}^{+\infty} \frac{1}{n^{\pi-2}}$ is divergent

2. $\int \left(1 - \frac{1}{x}\right)^2 \, dx =$

(a) $x - 2\ln|x| - \frac{2}{x^3} + C$

(b) $x - \frac{1}{x} - 2\ln|x| + C$

(c) $\frac{1}{3}\left(1 - \frac{1}{x}\right)^3 + C$

(d) $1 - \frac{1}{x} + C$

(e) $x + \frac{1}{x} - 2\ln|x| + C$

3. $\displaystyle\int_0^{\pi/2} \frac{\cos t}{1+\sin^2 t}\, dt =$

(a) 1

(b) $\ln 2$

(c) $\dfrac{\pi}{4}$

(d) 0

(e) $\dfrac{\pi}{3}$

4. The first four terms of the Taylor series of $f(x) = 4 + \ln x$ about $a = 1$ are given by

(a) $4 + x - \dfrac{1}{2}\, x^2 + \dfrac{1}{3}\, x^3$

(b) $4 + (x+1) - (x+1)^2 + 2(x+1)^3$

(c) $4 + (x-1) - \dfrac{1}{2}\,(x-1)^2 + \dfrac{1}{3}\,(x-1)^3$

(d) $4 + 5(x-1) - \dfrac{3}{2}\,(x-1)^2 + (x-1)^3$

(e) $4 + (x-1) - (x-1)^2 + 2(x-1)^3$

5. If f is a continuous function and $F(x) = \int_1^{x^3} f(\sqrt[3]{t})\, dt$, then $F'(x) =$

(a) $3x^2\, f(\sqrt[3]{x})$

(b) $\frac{1}{3}\, x^{-2/3}\, f(\sqrt[3]{x})$

(c) $\frac{1}{3x^2}\, f(x)$

(d) $3x^2\, f(x)$

(e) $f(x) - f(1)$

6. $\int \tan^3(2x) \sec^5(2x)\, dx =$

(a) $\frac{\tan^7(2x)}{14} - \frac{\tan^5(2x)}{10} + C$

(b) $\frac{\sec^7(2x)}{7} - \frac{\sec^5(2x)}{5} + C$

(c) $\frac{\sec^7(2x)}{14} - \frac{\sec^5(2x)}{10} + C$

(d) $\frac{\tan^4(2x)}{4} - \frac{\sec^6(2x)}{6} + C$

(e) $\sec^6(2x) - \sec^4(2x) + C$

7. $\displaystyle\int_{e}^{e^7} \frac{dx}{x\sqrt{2+2\ln x}}\, dx =$

(a) 10

(b) 2

(c) 6

(d) 4

(e) 8

8. $\displaystyle\int x \ln x \, dx =$

(a) $\dfrac{1}{2}\, x^2 \ln x - \dfrac{1}{2}\, x^2 + C$

(b) $\dfrac{1}{2}\, x^2 \ln x - \dfrac{1}{4}\, x^2 + C$

(c) $x \ln x + \dfrac{1}{2}\, x^2 + C$

(d) $\dfrac{1}{2}\, x^2 \ln x + \dfrac{1}{2}\, x(\ln x)^2 + C$

(e) $\dfrac{1}{2}\, (\ln x)^2 + C$

9. The area of the region bounded by the parabolas $y = (x+1)^2$ and $y^2 = x+1$ is equal to

(a) $\frac{2}{3}$

(b) $\frac{4}{3}$

(c) $\frac{1}{3}$

(d) 1

(e) $\frac{5}{3}$

10. The form of the partial fraction decomposition of $\dfrac{x^3+1}{x^2(x^2+4)^2}$ is

(a) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x+2)^2} + \dfrac{D}{(x-2)^2}$

(b) $\dfrac{A}{x^2} + \dfrac{B}{x^2+4} + \dfrac{Cx+D}{(x^2+4)^2}$

(c) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+4} + \dfrac{Ex+F}{(x^2+4)^2}$

(d) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^2+4} + \dfrac{D}{(x^2+4)^2}$

(e) $\dfrac{Ax+B}{x^2} + \dfrac{Cx+D}{(x^2+4)^2}$

11. The series $\sum_{n=1}^{+\infty} \frac{2^n + (-1)^{n-1}}{3^n}$

(a) diverges

(b) converges and its sum is $\frac{3}{4}$

(c) converges and its sum is 2

(d) converges and its sum is $\frac{2}{3}$

(e) converges and its sum is $\frac{9}{4}$

12. If the region enclosed by the curves $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$ is rotated about the y-axis, then the volume of the generated solid is

(a) $2\pi + 1$

(b) $\frac{\pi}{3}$

(c) 4π

(d) $\pi - 2$

(e) 2π

13. $\int \frac{x^4+x^2-1}{x^3+x}\,dx =$

(a) $\frac{1}{2}\,x^2 - \ln|x| + \frac{1}{2}\ln\ |x^3+x| + C$

(b) $\frac{1}{2}\,x^2 - \ln|x| + \frac{1}{2}\ln\ |x^2+1| + C$

(c) $\frac{1}{2}\,x^2 - \ln|x| + C$

(d) $\ln|x^4+x^2-1| - \frac{1}{2}\ \ln|x^3+x| + C$

(e) $\frac{1}{2}\,x^2 + \ln|x| - \frac{1}{x} + C$

14. Let R be the region in the first quadrant that is bounded by the curves $y=\sqrt[3]{x}$ and $y=x^3$. The volume of the solid obtained by rotating R about the line $y=1$ is given by

(a) $\pi\int_0^1 [(1-x^3)^2-(1-\sqrt[3]{x})^2]\,dx$

(b) $\pi\int_0^1 (y^{2/3}-y^2)\,dy$

(c) $2\pi\int_0^1 (1-x)(\sqrt[3]{x}-x^3)\,dx$

(d) $\pi\int_0^1 (x^3-\sqrt[3]{x})^2\,dx$

(e) $\pi\int_0^1 [(\sqrt[3]{y}-1)^2-(y^3-1)^2]\,dy$

15. The sequence $\left\{ \dfrac{(-1)^n n^2}{n^2 + n + 1} \right\}_{n=1}^{+\infty}$

(a) converges to 1

(b) diverges

(c) converges to -1

(d) converges to -2

(e) converges to 0

16. The improper integral $\displaystyle\int_0^2 \frac{1}{\sqrt[5]{x-1}}\, dx$

(a) converges and its value is $\dfrac{1}{4}$

(b) converges and its value is $\dfrac{5}{4}$

(c) converges and its value is $\dfrac{5}{2}$

(d) converges and its value is 0

(e) diverges

17. The length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{4}$, is

(a) $2 + \sqrt{2}$

(b) $\ln(1 + \sqrt{2})$

(c) $1 + \sqrt{2}$

(d) $\ln(\sqrt{2})$

(e) $\ln(\sqrt{2} + \sqrt{3})$

18. $\int \frac{x^2}{\sqrt{16 - x^2}}\, dx =$

(a) $8 \sin^{-1}\left(\frac{x}{4}\right) - \frac{1}{2}\, x\sqrt{16 - x^2} + C$

(b) $8 \sin^{-1}\left(\frac{x}{4}\right) - x + C$

(c) $\sqrt{16 - x^2} + C$

(d) $8 \sin^{-1}\left(\frac{x}{4}\right) - \sqrt{16 - x^2} + C$

(e) $4 \sin^{-1}\left(\frac{x}{4}\right) + 2x\sqrt{16 - x^2} + C$

19. The series $\sum_{n=1}^{+\infty}(-1)^n\frac{2^{2n}\cdot(n+1)^2}{n!}$ is

(a) convergent by the integral test

(b) a series with which the ratio test is inconclusive

(c) divergent by the ratio test

(d) convergent by the ratio test

(e) divergent by the test for divergence

20. The smallest number of terms of the series $\sum_{n=1}^{+\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ that we need to add so that $|\text{error}| < 0.1$ is

(a) 80

(b) 90

(c) 60

(d) 70

(e) 100

21. The value of the integral $\int_{-1}^{1} \frac{4 - x|x|}{2 + x}\, dx$ is equal to

(a) $5 + \ln 2$

(b) $-1 + 8 \ln 2$

(c) $-4 + \ln 2$

(d) $4 + 8 \ln 2$

(e) -2

22. The sum of the series $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{2^{4n+1}(2n)!}$

(a) is equal to $\frac{1}{2\sqrt{2}}$

(b) is equal to $\frac{1}{\sqrt{2}}$

(c) is equal to -1

(d) is equal to $\sqrt{2}$

(e) does not exist

23. The interval of convergence I and the radius of convergence R of the power series $\sum_{n=1}^{+\infty} (-1)^n \frac{x^n}{4^n \cdot n^3}$ are

(a) $I = [-3, 3)$, $R = 3$

(b) $I = (-4, 4]$, $R = 4$

(c) $I = (3, 4)$, $R = \frac{1}{2}$

(d) $I = [-4, 4]$, $R = 4$

(e) $I = (-4, 4)$, $R = 4$

24. If the power series $\sum_{n=0}^{+\infty} c_n(x+2)^n$ has a radius of convergence $R = 3$, then which one of the following is **TRUE**?

(a) $\sum_{n=0}^{+\infty} \frac{c_n}{2^n}$ is divergent

(b) $\sum_{n=0}^{+\infty} (-1)^n c_n 5^n$ is convergent

(c) $\sum_{n=0}^{+\infty} c_n 2^n$ is divergent

(d) $\sum_{n=0}^{+\infty} c_n 4^n$ is convergent

(e) $\sum_{n=0}^{+\infty} c_n$ is convergent

25. If the curve $x = \frac{1}{3}\sqrt{4-9y^2}\ , \quad 0 \le y \le \frac{1}{3},$ is rotated about the y-axis, then the area of the resulting surface is equal to

(a) $\frac{4\pi}{9}\sin^{-1}\left(\frac{2}{3}\right)$

(b) $\frac{2\pi}{3}\sin^{-1}\left(\frac{1}{3}\right)$

(c) $\frac{8\pi}{3}$

(d) $\frac{\pi}{9}$

(e) $\frac{4\pi}{9}$

26. The series $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ is

(a) divergent

(b) convergent by the comparison test

(c) convergent by the ratio test

(d) conditionally convergent

(e) absolutely convergent

27. The Maclaurin series for $f(x) = e^{-x^2/3}$ is given by

(a) $\sum_{n=0}^{+\infty} \frac{x^{2n}}{3^n \cdot n!}$

(b) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{3 \cdot n!}$

(c) $\sum_{n=1}^{+\infty} (-1)^n \frac{x^{n}}{3^n \cdot n!}$

(d) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{3^n \cdot n!}$

(e) $\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{x^{2n}}{9^n \cdot n!}$

28. The power series representation of $f(x) = \dfrac{x^2}{4+x^3}$ is

(a) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+3}}{4^{n+1}}$, $|x| < \sqrt[3]{4}$

(b) $\sum_{n=0}^{+\infty} \frac{x^{3n+2}}{4^{n}}$, $|x| < \sqrt[3]{4}$

(c) $\sum_{n=0}^{+\infty} (-1)^n \left(\frac{x}{4}\right)^{3n+2}$, $|x| < 4$

(d) $\sum_{n=0}^{+\infty} (-1)^n \frac{x^{3n+2}}{4^{n+1}}$, $|x| < \sqrt[3]{4}$

(e) $\sum_{n=0}^{+\infty} \frac{x^{n+3}}{4^{n+1}}$, $|x| < 4$

Q	MM	V1	V2	V3	V4
1	a	c	d	a	a
2	a	b	d	b	c
3	a	c	c	e	e
4	a	c	d	b	b
5	a	d	e	d	e
6	a	c	a	d	c
7	a	b	a	b	c
8	a	b	d	e	a
9	a	c	a	e	c
10	a	c	a	c	c
11	a	e	d	d	a
12	a	e	b	a	e
13	a	b	c	b	d
14	a	a	d	e	e
15	a	b	e	c	d
16	a	d	d	a	a
17	a	b	e	a	b
18	a	a	c	d	e
19	a	d	e	a	e
20	a	e	c	c	b
21	a	b	a	c	b
22	a	a	c	c	d
23	a	d	d	b	e
24	a	e	a	c	e
25	a	e	c	c	d
26	a	d	a	a	a
27	a	d	b	d	e
28	a	d	c	d	d