1. The value of the integral $\int_0^{\pi/4} \frac{\sin(2x)}{[1+\cos(2x)]^3}\,dx$ is

(a) $\frac{3}{16}$

(b) $\frac{1}{8}$

(c) $\frac{1}{16}$

(d) $\frac{1}{2}$

(e) 1

2. If $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$, then $\frac{dy}{dx} =$

(a) $\frac{3(1-3x)^3}{1+(1-3x)^2}$

(b) $\frac{-3(1-3x)^3}{1+(1-3x)^2}$

(c) $\frac{(1-3x)^3}{1+(1-3x)^2}$

(d) $\frac{27x^3}{1+9x^2}$

(e) $\frac{81x^3}{1+9x^2}$

3. The area of the region bounded by the graphs of $y = x^2 - 2$ and $y = x$ is

(a) $\frac{9}{2}$

(b) $\frac{3}{2}$

(c) $\frac{7}{2}$

(d) $\frac{5}{2}$

(e) $\frac{11}{2}$

4. The sum of the series $1 - \ln 3 + \frac{(\ln 3)^2}{2!} - \frac{(\ln 3)^3}{3!} + \cdots$

(a) is equal to $\frac{1}{3}$

(b) is equal to 3

(c) does not exist

(d) is equal to $e^{1/3}$

(e) is equal to e^3

5. The volume of the solid generated by rotating the region enclosed by the curves $y = x$ and $y = \sqrt{x}$ about the y-axis is

(a) $\pi \int_0^1 (y^2 - y^4) dy$

(b) $\pi \int_0^1 (y - y^2) dy$

(c) $\pi \int_0^1 (x^2 - x)\, dx$

(d) $\pi \int_{-1}^1 (y + y^2) dy$

(e) $\pi \int_{-1}^0 (x - x^2)\, dx$

6. The sequence $\{(2 - e)^n\}_{n=1}^{+\infty}$

(a) converges to 0

(b) converges to $-e$

(c) converges to $\frac{2}{e}$

(d) converges to 2

(e) diverges

7. If the n-th partial sum of a series $\sum_{n=1}^{+\infty} a_n$ is $s_n = 2 - \frac{(-1)^n}{n^2}$, then the series $\sum_{n=1}^{+\infty} a_n$

(a) converges and its sum is 2

(b) converges and its sum is 1

(c) diverges

(d) converges and its sum is $\frac{3}{2}$

(e) converges and its sum is $\frac{1}{2}$

8. The series $\sum_{n=1}^{+\infty} \frac{(-3)^{n+1}}{2^{3n}}$

(a) converges and its sum is $\frac{9}{11}$

(b) converges and its sum is $\frac{9}{5}$

(c) converges and its sum is $\frac{-24}{11}$

(d) converges and its sum is $\frac{-3}{11}$

(e) diverges

9. The series $1 + \dfrac{1}{2^2\sqrt{2}} + \dfrac{1}{3^2\sqrt{3}} + \dfrac{1}{4^2\sqrt{4}} + \cdots$ is

(a) a convergent p-series with $p = \dfrac{5}{2}$

(b) a divergent series

(c) a convergent p-series with $p = 2$

(d) a divergent series by the integral test

(e) a divergent p-series with $p = \dfrac{1}{2}$

10. Suppose that $f(1) = 1, f(4) = 7, f'(1) = -1, f'(4) = 3$, and f'' is continuous. Then the value of $\int_1^4 x f''(x)\, dx$ is equal to [Hint: Use integration by parts]

(a) 7

(b) 2

(c) 5

(d) 12

(e) 0

11. The average value of the function $f(x) = \dfrac{x}{(x+3)^3}$ over the interval $[-1, 1]$ is

(a) $\dfrac{-1}{64}$

(b) $\dfrac{3}{32}$

(c) $\dfrac{-5}{32}$

(d) $\dfrac{5}{64}$

(e) 0

12. The series $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n \ln n}$

(a) diverges by the integral test

(b) converges by the integral test

(c) converges by the comparison test with $b_n = \dfrac{1}{n}$

(d) diverges by the comparison test with $b_n = \dfrac{1}{n^2}$

(e) diverges by the ratio test

13. The error in approximating the sum of the series $\sum_{n=1}^{+\infty} \frac{(-1)^n n}{5^n}$ by the sum of the first four terms is less than or equal to

(a) $\frac{1}{5^4}$

(b) $\frac{1}{4 \cdot 5^4}$

(c) $\frac{6}{5^6}$

(d) $\frac{1}{5^5}$

(e) $\frac{4}{5^5}$

14. The length of the curve $y = \ln(\sec x), \quad 0 \leq x \leq \frac{\pi}{4}$, is

(a) $\ln(1 + \sqrt{2})$

(b) $\ln(\sqrt{2})$

(c) $1 + \sqrt{2}$

(d) $\ln(\sqrt{2} + \sqrt{3})$

(e) $2 + \sqrt{2}$

15. The improper integral $\int_0^2 \frac{x^3}{\sqrt{4-x^2}}\,dx$

(a) has the value $\frac{16}{3}$

(b) has the value $\frac{22}{3}$

(c) has the value $\frac{11}{3}$

(d) has the value $\frac{19}{3}$

(e) is divergent

16. The integral $\int \frac{e^{-x}}{e^{-2x}+3e^{-x}+2}\,dx$ equals

(a) $\ln\left(\frac{2+e^{-x}}{1+e^{-x}}\right)+C$

(b) $\ln\left(\frac{2+e^{-x}}{1+e^{x}}\right)+C$

(c) $\ln\left(\frac{2-e^{-x}}{1-e^{-x}}\right)+C$

(d) $\ln(2+e^{-x})+\ln(1+e^{-x})+C$

(e) $\ln(2-e^{-x})+\ln(1-e^{-x})+C$

17. The value of the integral $\int_1^{16} \frac{1}{\sqrt{x}+\sqrt[4]{x}}\,dx$ is equal to

(a) $2+4\ln(1.5)$

(b) $3-\ln 16$

(c) $2-4\ln 3$

(d) $4+\ln(1.5)$

(e) $\ln(81)$

18. The series $\sum_{n=1}^{+\infty} n\sin\left(\frac{1}{n}\right)$

(a) diverges

(b) converges and its sum is 1

(c) converges and its sum is 0

(d) converges

(e) converges and its sum is $\frac{1}{3}$

19. The series $\sum_{n=1}^{+\infty} \frac{n^2+1}{n^5+n^4+1}$ is

(a) convergent

(b) divergent

(c) convergent and its sum is 1

(d) divergent by the test of divergence

(e) convergent by the ratio test

20. The series $\sum_{n=1}^{+\infty} \frac{(-1)^n 3n}{4n-1}$ is

(a) divergent

(b) convergent

(c) absolutely convergent

(d) conditionally convergent

(e) neither convergent nor divergent

21. The integral for the area of the surface obtained by rotating the curve $y = \tan x$ from $(0, 0)$ to $\left(\frac{\pi}{4}, 1\right)$ about the y-axis is

(a) $2\pi \int_0^{\pi/4} x\sqrt{1 + \sec^4 x}\, dx$

(b) $2\pi \int_0^{\pi/4} x\sqrt{1 + \tan^4 x}\, dx$

(c) $2\pi \int_0^{\pi/4} \tan x\sqrt{1 + \sec^4 x}\, dx$

(d) $2\pi \int_0^{1} y\sqrt{1 + \frac{1}{1+y^2}}dy$

(e) $2\pi \int_0^{\pi/4} \tan x\sqrt{1 - \tan^2 x}\, dx$

22. The area of the region between the x-axis and the curve $y = \frac{x}{e^x}$ for $x \geq 0$ is

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) 3

23. $\int_{1/2}^{3/2} \frac{dx}{5-4x+4x^2}\, dx =$

(a) $\frac{\pi}{16}$

(b) $\frac{3\pi}{16}$

(c) $\frac{3\pi}{4}$

(d) $\frac{5\pi}{8}$

(e) $\frac{3\pi}{8}$

24. The series $\sum_{n=1}^{+\infty} \left(\frac{1+\ln n}{n^2+3}\right)^n$ is

(a) convergent by the root test

(b) divergent by the root test

(c) a convergent geometric series

(d) a series with which the root test is inconclusive

(e) divergent by the test of divergence

25. The interval of convergence and the radius of convergence R of the power series $\sum_{n=0}^{+\infty} \frac{(-3)^{n+1}(2x+1)^n}{\sqrt{n+1}}$ are

(a) $\left(\frac{-2}{3}, \frac{-1}{3}\right]$; $R = \frac{1}{6}$

(b) $\left(\frac{-2}{3}, \frac{-1}{3}\right)$; $R = \frac{2}{9}$

(c) $\left[\frac{-2}{3}, \frac{1}{3}\right]$; $R = \frac{1}{6}$

(d) $\left(\frac{-2}{3}, \frac{-1}{3}\right]$; $R = \frac{1}{9}$

(e) $\left(\frac{-2}{3}, \frac{1}{3}\right]$; $R = \frac{1}{6}$

26. The value of the integral $\int_0^{1/3} \frac{x^2}{1+x^7}\, dx$ is equal to

(a) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+3) \cdot 3^{7n+3}}$

(b) $\sum_{n=0}^{+\infty} \frac{(-1)^n \cdot 3^{7n+3}}{7n+3}$

(c) $\sum_{n=0}^{+\infty} \frac{(-1)^n}{(7n+1) \cdot 3^{7n+1}}$

(d) $\sum_{n=0}^{+\infty} \frac{1}{(7n+1) \cdot 3^{7n+3}}$

(e) $\sum_{n=1}^{+\infty} \frac{(-1)^n(7n+3)}{3^{7n+1}}$

27. If the region bounded by the curves $y = \sqrt{x-1}, y = 0$, and $x = 5$ is rotated about the line $y = 3$, then the volume of the generated solid is

(a) 24π

(b) 10π

(c) 6π

(d) 36π

(e) 4π

28. The Maclaurin series of $f(x) = x\cos(x^3)$ is

(a) $\sum_{n=0}^{+\infty}(-1)^n \frac{x^{6n+1}}{(2n)!}$

(b) $\sum_{n=0}^{+\infty} \frac{x^{6n}}{(2n)!}$

(c) $\sum_{n=0}^{+\infty}(-1)^n \frac{x^{6n+1}}{(6n+1)!}$

(d) $\sum_{n=0}^{+\infty}(-1)^n \frac{x^{3n+1}}{(2n)!}$

(e) $\sum_{n=0}^{+\infty}(-1)^n \frac{x^{5n+1}}{(2n)!}$