1. **Using the method of cylindrical shells**, set up, but do not evaluate, an integral for the volume of the solid obtained by rotating

   (a) the region bounded by the curves $y = x - x^2$ and $y = 0$ about the line $x = -1$. Sketch the region and a typical rectangle. (5 points)

   (b) the region bounded by the curves $y = \ln x, y = 0$, and $x = 2$ about the x-axis. Sketch the region and a typical rectangle. (5 points)

2. Find all numbers b such that the average value of $f(x) = \sqrt{x}$ on the interval $[0, b]$ is 6. (7 points)

3. Evaluate the following integrals:

(a) $\int \tan^2 t \sec^4 t \, dt.$ (7 points)

(b) $\int e^{-x} \cos(2x) \, dx.$ (7 points)

(c) $\displaystyle\int \frac{1}{x^3\sqrt{x^2-1}}\,dx.$ (7 points)

(d) $\displaystyle\int x^3\,e^{x^2}\,dx.$ (7 points)

(e) $\displaystyle\int \frac{1}{5+3\cos x}\,dx$ (7 points)

(f) $\displaystyle\int \frac{1}{(-x^2-2x)^{3/2}}\,dx.$ (7 points)

4. (a) Write out the form of the partial fraction decomposition of $\dfrac{x-2}{x(x^3+x)^2}$. Do not determine the numerical values of the coefficients. (5 points)

(b) Evaluate $\displaystyle\int \frac{x^3-4x-10}{x^2-x-6}\,dx$. (8 points)

5. Evaluate the integral or show that it is divergent.

(a) $\displaystyle\int_{-\infty}^{0} \frac{3x}{(5x^2+6)^2}\,dx.$ (7 points)

(b) $\displaystyle\int_{0}^{2} \frac{1}{(x-1)^3}\,dx.$ (7 points)

6. (a) Find the arc length of the curve $y = x^2 - \frac{1}{8}\ln x, \quad 1 \leq x \leq 3.$ (7 points)

(b) Find the area of the surface generated by rotating the curve $y = \sqrt{4 - x^2}, \quad 0 \leq x \leq 1$ about the y-axis. (7 points)