

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Summer (073)
Tuesday 26/8/2008

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5 per question

This exam was prepared using mcqs
For questions send an email to Dr. Ibrahim Al-Lehyani (iallehyani@kaau.edu.sa)

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
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Tuesday 26/8/2008
Net Time Allowed: 180 minutes

MASTER VERSION

1. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

(e) $\frac{2}{5}$

2. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the y -axis, then the volume of the solid generated is equal to

(a) $2\pi^2$

(b) π^2

(c) $\frac{\pi^2}{2}$

(d) $\frac{\pi^2}{4}$

(e) $4\pi^2$

3. The series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 1}{2n^3 + n} \right)^n$

- (a) Converges by the Root test
- (b) Diverges by the Root test
- (c) Diverges by the Ratio test
- (d) is a convergent geometric series
- (e) Diverges by the limit comparison test

4. $\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$

- (a) $\frac{27}{4}$
- (b) $\frac{9}{4}$
- (c) $\frac{11}{4}$
- (d) $\frac{31}{4}$
- (e) $\frac{23}{4}$

5. $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} dx =$

(a) $x + \sqrt{1-x^2} + c$

(b) $x - \sin^{-1} x + c$

(c) $x - \frac{1}{2}\sqrt{1-x^2} + c$

(d) $x + \sin^{-1} x + c$

(e) $x + \frac{3}{2}\sqrt{1-x^2} + c$

6. $\int_0^\pi |\sin 2x| dx =$

(a) 2

(b) 0

(c) -1

(d) 1

(e) $\frac{1}{2}$

7. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left(\frac{1}{5}, \infty\right)$

(b) $(1, \infty)$

(c) $[1, \infty)$

(d) $\left[\frac{1}{5}, \infty\right)$

(e) $(0, \infty)$

8. The area of the surface obtained by rotating the curve $y = \sqrt{3-x^2}$, $0 \leq x \leq 1$, about the x -axis is equal to

(a) $2\pi\sqrt{3}$

(b) $\frac{4\pi\sqrt{3}}{3}$

(c) $\frac{4\sqrt{3}}{2}$

(d) $6\pi\sqrt{3}$

(e) $\frac{\pi\sqrt{3}}{6}$

9. $\int \tan^{-1} \left(\frac{1}{x} \right) dx =$

(a) $x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

(b) $\frac{1}{2} x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

(c) $(x+1) \tan^{-1} \left(\frac{1}{x} \right) + c$

(d) $x \tan^{-1} \left(\frac{1}{x} \right) - \ln(1+x^2) + c$

(e) $\left(\frac{1}{x^2} \right) \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

10. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4+5}}$

(a) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

(b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(c) Converges by the integral test

(d) Diverges by the test for divergence

(e) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

11. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}} dx =$

(a) $\frac{4}{3}$

(b) $\frac{7}{3}$

(c) $\frac{\sqrt{3}}{3}$

(d) $\frac{1}{3}$

(e) $\sqrt{3}$

12. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (2x-3)^n}{n \cdot 4^n}$ are given by

(a) $R = 2, I = \left(-\frac{1}{2}, \frac{7}{2}\right]$

(b) $R = 2, I = [-4, 4]$

(c) $R = 2, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

(d) $R = 4, I = (-4, 4]$

(e) $R = 4, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

13. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{7\pi}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{9\pi}{2}$

(d) π

(e) $\frac{5\pi}{2}$

14. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclaurin series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{29}{6}$

(b) $\frac{7}{3}$

(c) $\frac{5}{6}$

(d) $\frac{14}{3}$

(e) $\frac{31}{6}$

15. The sequence $\left\{ (2n + 1) \sin \frac{7}{n} \right\}$

(a) Converges to 14

(b) Converges to $\frac{7}{2}$

(c) Converges to 0

(d) Converges to $\frac{2}{7}$

(e) Diverges

16. If $x > e$, then $\frac{d}{dx} \int_1^{\sqrt{\ln x}} e^{t^2} dt =$

(a) $\frac{1}{2\sqrt{\ln x}}$

(b) $\frac{4}{\sqrt{\ln x}}$

(c) 0

(d) x

(e) $2x$

17. $\int \frac{x^2 + x + 3}{(x - 1)(x^2 + 2x + 2)} dx =$

- (a) $\ln|x - 1| - \tan^{-1}(x + 1) + c$
- (b) $x + \ln|x - 1| + \tan^{-1}(x + 1) + c$
- (c) $\ln(x - 1)^2 - \tan^{-1}(x + 1) + c$
- (d) $2x + \ln|x - 1| - 3 \tan^{-1}(x + 1) + c$
- (e) $\ln|x - 1| + 2 \tan^{-1}(x + 1) + c$

18. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5 6^n}{(n + 1)!}$

- (a) is absolutely convergent
- (b) is conditionally convergent
- (c) is divergent by the ratio test
- (d) is divergent by the test for divergence
- (e) is convergent by the integral test

19. The value of the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \left(\cos \frac{i\pi}{2n} \right)^2$ on the interval $\left[0, \frac{\pi}{2} \right]$ is

(a) $\frac{\pi}{8}$

(b) $\frac{\pi}{2}$

(c) $1 + \frac{\pi}{8}$

(d) $1 + \frac{\pi}{2}$

(e) $-\frac{1}{4} + \frac{\pi}{8}$

20. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges conditionally

(b) Converges absolutely

(c) Diverges

(d) Converges by the integral test

(e) Converges by the root test

21. $\int_0^{\frac{\pi}{4}} \sqrt{\frac{1 + \sin x}{1 - \sin x}} dx =$

(a) $\ln(2 + \sqrt{2})$

(b) $\ln(2\sqrt{2} - 1)$

(c) $\ln \sqrt{2}$

(d) $\ln(1 + 2\sqrt{2})$

(e) $\ln(1 + \sqrt{2})$

22. The improper integral $\int_{-\infty}^{\infty} e^{-|x|} dx$

(a) Converges to 2

(b) Converges to $\frac{1}{2}$

(c) Converges to 1

(d) Converges to 0

(e) Diverges

23. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} - \frac{2}{3}$

(b) $\frac{\pi}{4} - \frac{1}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4} + \frac{1}{3}$

(e) $\frac{\pi}{4} + \frac{4}{3}$

24. The improper integral $\int_0^1 \frac{1}{e^x - 1} dx$

(a) Diverges

(b) Converges to $\ln(e - 1)$

(c) Converges to $\ln(1 - e^{-1})$

(d) Converges to 1

(e) Converges to 0

25. If $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$, then $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{16}$

(d) π

(e) π^2

26. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 499

(b) 599

(c) 488

(d) 198

(e) 408

27. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50 \cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

- (a) $50(e^2 - e^{-2})$
- (b) $100(e^2 + e^{-2})$
- (c) $50(e - e^{-1})$
- (d) $100(e + e^{-1})$
- (e) $50(e^2 - e^{-2})^2$
28. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $0 < x < \pi$, we get
- $$\int \frac{2 dx}{\sin x(1 + \cos x)} = A \ln\left(\tan \frac{x}{2}\right) + B \tan^2 \frac{x}{2} + c \text{ where } A + 2B =$$
- (a) 2
- (b) 1
- (c) 3
- (d) 0
- (e) 4

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Department of Mathematics and Statistics

CODE 001

Math 102

CODE 001

Final Exam

Summer (073)

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Net Time Allowed: 180 minutes

Name: _____

ID: _____ Sec: _____

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the y -axis, then the volume of the solid generated is equal to

(a) π^2

(b) $\frac{\pi^2}{4}$

(c) $4\pi^2$

(d) $\frac{\pi^2}{2}$

(e) $2\pi^2$

2. The series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 1}{2n^3 + n} \right)^n$

(a) Diverges by the limit comparison test

(b) Converges by the Root test

(c) is a convergent geometric series

(d) Diverges by the Root test

(e) Diverges by the Ratio test

3. $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} dx =$

(a) $x + \sqrt{1-x^2} + c$

(b) $x + \sin^{-1} x + c$

(c) $x - \sin^{-1} x + c$

(d) $x - \frac{1}{2}\sqrt{1-x^2} + c$

(e) $x + \frac{3}{2}\sqrt{1-x^2} + c$

4. $\int_0^\pi |\sin 2x| dx =$

(a) $\frac{1}{2}$

(b) 1

(c) -1

(d) 0

(e) 2

5. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{2}{5}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) $\frac{1}{3}$

(e) $\frac{2}{3}$

6. $\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$

(a) $\frac{31}{4}$

(b) $\frac{9}{4}$

(c) $\frac{11}{4}$

(d) $\frac{27}{4}$

(e) $\frac{23}{4}$

7. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{9\pi}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{7\pi}{2}$

(d) π

(e) $\frac{5\pi}{2}$

8. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges conditionally

(b) Converges by the root test

(c) Diverges

(d) Converges by the integral test

(e) Converges absolutely

9. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5 6^n}{(n+1)!}$
- (a) is divergent by the test for divergence
 - (b) is conditionally convergent
 - (c) is convergent by the integral test
 - (d) is divergent by the ratio test
 - (e) is absolutely convergent
10. The area of the surface obtained by rotating the curve $y = \sqrt{3-x^2}$, $0 \leq x \leq 1$, about the x -axis is equal to
- (a) $\frac{4\sqrt{3}}{2}$
 - (b) $2\pi\sqrt{3}$
 - (c) $6\pi\sqrt{3}$
 - (d) $\frac{4\pi\sqrt{3}}{3}$
 - (e) $\frac{\pi\sqrt{3}}{6}$

11. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclaurin series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{5}{6}$

(b) $\frac{14}{3}$

(c) $\frac{29}{6}$

(d) $\frac{7}{3}$

(e) $\frac{31}{6}$

12. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (2x - 3)^n}{n \cdot 4^n}$ are given by

(a) $R = 2, I = [-4, 4]$

(b) $R = 4, I = (-4, 4]$

(c) $R = 2, I = \left(-\frac{1}{2}, \frac{7}{2}\right]$

(d) $R = 2, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

(e) $R = 4, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

13. The sequence $\left\{ (2n + 1) \sin \frac{7}{n} \right\}$

(a) Converges to 0

(b) Converges to $\frac{7}{2}$

(c) Converges to $\frac{2}{7}$

(d) Diverges

(e) Converges to 14

14. If $x > e$, then $\frac{d}{dx} \int_1^{\sqrt{\ln x}} e^{t^2} dt =$

(a) $2x$

(b) 0

(c) $\frac{4}{\sqrt{\ln x}}$

(d) x

(e) $\frac{1}{2\sqrt{\ln x}}$

15. The value of the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \left(\cos \frac{i\pi}{2n} \right)^2$ on the interval $\left[0, \frac{\pi}{2} \right]$ is

(a) $1 + \frac{\pi}{8}$

(b) $-\frac{1}{4} + \frac{\pi}{8}$

(c) $\frac{\pi}{8}$

(d) $\frac{\pi}{2}$

(e) $1 + \frac{\pi}{2}$

16. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}} dx =$

(a) $\frac{1}{3}$

(b) $\sqrt{3}$

(c) $\frac{4}{3}$

(d) $\frac{7}{3}$

(e) $\frac{\sqrt{3}}{3}$

17. $\int \tan^{-1} \left(\frac{1}{x} \right) dx =$

(a) $x \tan^{-1} \left(\frac{1}{x} \right) - \ln(1 + x^2) + c$

(b) $\left(\frac{1}{x^2} \right) \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(c) $\frac{1}{2} x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(d) $x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(e) $(x + 1) \tan^{-1} \left(\frac{1}{x} \right) + c$

18. $\int_0^{\frac{\pi}{4}} \sqrt{\frac{1 + \sin x}{1 - \sin x}} dx =$

(a) $\ln \sqrt{2}$

(b) $\ln(1 + \sqrt{2})$

(c) $\ln(2\sqrt{2} - 1)$

(d) $\ln(2 + \sqrt{2})$

(e) $\ln(1 + 2\sqrt{2})$

19. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4 + 5}}$

(a) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

(b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

(c) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(d) Diverges by the test for divergence

(e) Converges by the integral test

20. The improper integral $\int_{-\infty}^{\infty} e^{-|x|} dx$

(a) Converges to $\frac{1}{2}$

(b) Converges to 0

(c) Converges to 1

(d) Converges to 2

(e) Diverges

21. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left[\frac{1}{5}, \infty\right)$

(b) $\left(\frac{1}{5}, \infty\right)$

(c) $[1, \infty)$

(d) $(1, \infty)$

(e) $(0, \infty)$

22. $\int \frac{x^2 + x + 3}{(x-1)(x^2 + 2x + 2)} dx =$

(a) $\ln(x-1)^2 - \tan^{-1}(x+1) + c$

(b) $\ln|x-1| + 2\tan^{-1}(x+1) + c$

(c) $x + \ln|x-1| + \tan^{-1}(x+1) + c$

(d) $\ln|x-1| - \tan^{-1}(x+1) + c$

(e) $2x + \ln|x-1| - 3\tan^{-1}(x+1) + c$

23. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50 \cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

- (a) $50(e^2 - e^{-2})$
(b) $100(e^2 + e^{-2})$
(c) $50(e^2 - e^{-2})^2$
(d) $50(e - e^{-1})$
(e) $100(e + e^{-1})$
24. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $0 < x < \pi$, we get

$$\int \frac{2 dx}{\sin x(1 + \cos x)} = A \ln\left(\tan \frac{x}{2}\right) + B \tan^2 \frac{x}{2} + c \text{ where } A + 2B =$$

- (a) 1
(b) 2
(c) 3
(d) 0
(e) 4

25. The improper integral $\int_0^1 \frac{1}{e^x - 1} dx$

(a) Converges to $\ln(1 - e^{-1})$

(b) Converges to 1

(c) Converges to 0

(d) Diverges

(e) Converges to $\ln(e - 1)$

26. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} - \frac{1}{3}$

(b) $\frac{\pi}{4} - \frac{2}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4} + \frac{1}{3}$

(e) $\frac{\pi}{4} + \frac{4}{3}$

27. If $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$, then $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$

(a) π^2

(b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{16}$

(d) $\frac{\pi}{2}$

(e) π

28. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 499

(b) 488

(c) 599

(d) 408

(e) 198

Name

ID

Sec

1	a	b	c	d	e	f
2	a	b	c	d	e	f
3	a	b	c	d	e	f
4	a	b	c	d	e	f
5	a	b	c	d	e	f
6	a	b	c	d	e	f
7	a	b	c	d	e	f
8	a	b	c	d	e	f
9	a	b	c	d	e	f
10	a	b	c	d	e	f
11	a	b	c	d	e	f
12	a	b	c	d	e	f
13	a	b	c	d	e	f
14	a	b	c	d	e	f
15	a	b	c	d	e	f
16	a	b	c	d	e	f
17	a	b	c	d	e	f
18	a	b	c	d	e	f
19	a	b	c	d	e	f
20	a	b	c	d	e	f
21	a	b	c	d	e	f
22	a	b	c	d	e	f
23	a	b	c	d	e	f
24	a	b	c	d	e	f
25	a	b	c	d	e	f
26	a	b	c	d	e	f
27	a	b	c	d	e	f
28	a	b	c	d	e	f
29	a	b	c	d	e	f
30	a	b	c	d	e	f
31	a	b	c	d	e	f
32	a	b	c	d	e	f
33	a	b	c	d	e	f
34	a	b	c	d	e	f
35	a	b	c	d	e	f

36	a	b	c	d	e	f
37	a	b	c	d	e	f
38	a	b	c	d	e	f
39	a	b	c	d	e	f
40	a	b	c	d	e	f
41	a	b	c	d	e	f
42	a	b	c	d	e	f
43	a	b	c	d	e	f
44	a	b	c	d	e	f
45	a	b	c	d	e	f
46	a	b	c	d	e	f
47	a	b	c	d	e	f
48	a	b	c	d	e	f
49	a	b	c	d	e	f
50	a	b	c	d	e	f
51	a	b	c	d	e	f
52	a	b	c	d	e	f
53	a	b	c	d	e	f
54	a	b	c	d	e	f
55	a	b	c	d	e	f
56	a	b	c	d	e	f
57	a	b	c	d	e	f
58	a	b	c	d	e	f
59	a	b	c	d	e	f
60	a	b	c	d	e	f
61	a	b	c	d	e	f
62	a	b	c	d	e	f
63	a	b	c	d	e	f
64	a	b	c	d	e	f
65	a	b	c	d	e	f
66	a	b	c	d	e	f
67	a	b	c	d	e	f
68	a	b	c	d	e	f
69	a	b	c	d	e	f
70	a	b	c	d	e	f

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 002

Math 102

CODE 002

Final Exam

Summer (073)

Tuesday 26/8/2008

Net Time Allowed: 180 minutes

Name: _____

ID: _____ Sec: _____

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1.
$$\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$$

(a) $\frac{27}{4}$

(b) $\frac{9}{4}$

(c) $\frac{31}{4}$

(d) $\frac{11}{4}$

(e) $\frac{23}{4}$

2.
$$\int_0^{\pi} |\sin 2x| \, dx =$$

(a) 2

(b) $\frac{1}{2}$

(c) 0

(d) 1

(e) -1

3. $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} dx =$

(a) $x - \sin^{-1} x + c$

(b) $x + \sqrt{1-x^2} + c$

(c) $x - \frac{1}{2}\sqrt{1-x^2} + c$

(d) $x + \sin^{-1} x + c$

(e) $x + \frac{3}{2}\sqrt{1-x^2} + c$

4. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{2}{5}$

(b) $\frac{3}{4}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

(e) $\frac{1}{2}$

5. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the y -axis, then the volume of the solid generated is equal to

(a) $\frac{\pi^2}{4}$

(b) π^2

(c) $2\pi^2$

(d) $4\pi^2$

(e) $\frac{\pi^2}{2}$

6. The series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 1}{2n^3 + n} \right)^n$

- (a) Converges by the Root test
- (b) Diverges by the Ratio test
- (c) Diverges by the limit comparison test
- (d) is a convergent geometric series
- (e) Diverges by the Root test

7. $\int_0^{\frac{\pi}{4}} \sqrt{\frac{1 + \sin x}{1 - \sin x}} dx =$

(a) $\ln(2 + \sqrt{2})$

(b) $\ln(1 + 2\sqrt{2})$

(c) $\ln(1 + \sqrt{2})$

(d) $\ln \sqrt{2}$

(e) $\ln(2\sqrt{2} - 1)$

8. The area of the surface obtained by rotating the curve $y = \sqrt{3 - x^2}$, $0 \leq x \leq 1$, about the x -axis is equal to

(a) $\frac{\pi\sqrt{3}}{6}$

(b) $\frac{4\pi\sqrt{3}}{3}$

(c) $6\pi\sqrt{3}$

(d) $2\pi\sqrt{3}$

(e) $\frac{4\sqrt{3}}{2}$

9. The sequence $\left\{ (2n + 1) \sin \frac{7}{n} \right\}$

(a) Converges to $\frac{7}{2}$

(b) Diverges

(c) Converges to $\frac{2}{7}$

(d) Converges to 14

(e) Converges to 0

10. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4 + 5}}$

(a) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

(c) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

(d) Diverges by the test for divergence

(e) Converges by the integral test

11. The improper integral $\int_{-\infty}^{\infty} e^{-|x|} dx$

(a) Converges to 1

(b) Converges to $\frac{1}{2}$

(c) Converges to 0

(d) Converges to 2

(e) Diverges

12. If $x > e$, then $\frac{d}{dx} \int_1^{\sqrt{\ln x}} e^{t^2} dt =$

(a) 0

(b) x

(c) $2x$

(d) $\frac{1}{2\sqrt{\ln x}}$

(e) $\frac{4}{\sqrt{\ln x}}$

13. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (2x - 3)^n}{n \cdot 4^n}$ are given by

(a) $R = 2, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

(b) $R = 2, I = [-4, 4]$

(c) $R = 2, I = \left(-\frac{1}{2}, \frac{7}{2}\right]$

(d) $R = 4, I = (-4, 4]$

(e) $R = 4, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

14. $\int \tan^{-1}\left(\frac{1}{x}\right) dx =$

(a) $(x + 1) \tan^{-1}\left(\frac{1}{x}\right) + c$

(b) $\frac{1}{2} x \tan^{-1}\left(\frac{1}{x}\right) + \ln \sqrt{1 + x^2} + c$

(c) $x \tan^{-1}\left(\frac{1}{x}\right) - \ln(1 + x^2) + c$

(d) $x \tan^{-1}\left(\frac{1}{x}\right) + \ln \sqrt{1 + x^2} + c$

(e) $\left(\frac{1}{x^2}\right) \tan^{-1}\left(\frac{1}{x}\right) + \ln \sqrt{1 + x^2} + c$

15. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left[\frac{1}{5}, \infty\right)$

(b) $\left(\frac{1}{5}, \infty\right)$

(c) $[1, \infty)$

(d) $(0, \infty)$

(e) $(1, \infty)$

16. $\int \frac{x^2 + x + 3}{(x-1)(x^2 + 2x + 2)} dx =$

(a) $x + \ln|x-1| + \tan^{-1}(x+1) + c$

(b) $\ln|x-1| + 2\tan^{-1}(x+1) + c$

(c) $\ln(x-1)^2 - \tan^{-1}(x+1) + c$

(d) $2x + \ln|x-1| - 3\tan^{-1}(x+1) + c$

(e) $\ln|x-1| - \tan^{-1}(x+1) + c$

17. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{\pi}{2}$

(b) $\frac{7\pi}{2}$

(c) $\frac{9\pi}{2}$

(d) π

(e) $\frac{5\pi}{2}$

18. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges absolutely

(b) Converges by the root test

(c) Converges by the integral test

(d) Diverges

(e) Converges conditionally

19. The value of the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \left(\cos \frac{i\pi}{2n} \right)^2$ on the interval $\left[0, \frac{\pi}{2} \right]$ is

(a) $1 + \frac{\pi}{2}$

(b) $1 + \frac{\pi}{8}$

(c) $-\frac{1}{4} + \frac{\pi}{8}$

(d) $\frac{\pi}{8}$

(e) $\frac{\pi}{2}$

20. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5 6^n}{(n+1)!}$

(a) is convergent by the integral test

(b) is divergent by the test for divergence

(c) is absolutely convergent

(d) is divergent by the ratio test

(e) is conditionally convergent

21. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclaurin series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{31}{6}$

(b) $\frac{5}{6}$

(c) $\frac{29}{6}$

(d) $\frac{14}{3}$

(e) $\frac{7}{3}$

22. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}} dx =$

(a) $\frac{1}{3}$

(b) $\frac{\sqrt{3}}{3}$

(c) $\frac{7}{3}$

(d) $\sqrt{3}$

(e) $\frac{4}{3}$

23. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 499

(b) 488

(c) 599

(d) 198

(e) 408

24. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} + \frac{4}{3}$

(b) $\frac{\pi}{4} - \frac{2}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4} + \frac{1}{3}$

(e) $\frac{\pi}{4} - \frac{1}{3}$

25. The improper integral $\int_0^1 \frac{1}{e^x - 1} dx$

- (a) Converges to 1
- (b) Converges to $\ln(1 - e^{-1})$
- (c) Converges to 0
- (d) Diverges
- (e) Converges to $\ln(e - 1)$

26. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $0 < x < \pi$, we get

$$\int \frac{2 dx}{\sin x(1 + \cos x)} = A \ln\left(\tan \frac{x}{2}\right) + B \tan^2 \frac{x}{2} + c \text{ where } A + 2B =$$

- (a) 2
- (b) 1
- (c) 4
- (d) 0
- (e) 3

27. If $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$, then $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi^2}{16}$

(c) π^2

(d) π

(e) $\frac{\pi}{2}$

28. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50 \cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

(a) $50(e - e^{-1})$

(b) $100(e + e^{-1})$

(c) $50(e^2 - e^{-2})$

(d) $100(e^2 + e^{-2})$

(e) $50(e^2 - e^{-2})^2$

Name

ID

Sec

1	a	b	c	d	e	f
2	a	b	c	d	e	f
3	a	b	c	d	e	f
4	a	b	c	d	e	f
5	a	b	c	d	e	f
6	a	b	c	d	e	f
7	a	b	c	d	e	f
8	a	b	c	d	e	f
9	a	b	c	d	e	f
10	a	b	c	d	e	f
11	a	b	c	d	e	f
12	a	b	c	d	e	f
13	a	b	c	d	e	f
14	a	b	c	d	e	f
15	a	b	c	d	e	f
16	a	b	c	d	e	f
17	a	b	c	d	e	f
18	a	b	c	d	e	f
19	a	b	c	d	e	f
20	a	b	c	d	e	f
21	a	b	c	d	e	f
22	a	b	c	d	e	f
23	a	b	c	d	e	f
24	a	b	c	d	e	f
25	a	b	c	d	e	f
26	a	b	c	d	e	f
27	a	b	c	d	e	f
28	a	b	c	d	e	f
29	a	b	c	d	e	f
30	a	b	c	d	e	f
31	a	b	c	d	e	f
32	a	b	c	d	e	f
33	a	b	c	d	e	f
34	a	b	c	d	e	f
35	a	b	c	d	e	f

36	a	b	c	d	e	f
37	a	b	c	d	e	f
38	a	b	c	d	e	f
39	a	b	c	d	e	f
40	a	b	c	d	e	f
41	a	b	c	d	e	f
42	a	b	c	d	e	f
43	a	b	c	d	e	f
44	a	b	c	d	e	f
45	a	b	c	d	e	f
46	a	b	c	d	e	f
47	a	b	c	d	e	f
48	a	b	c	d	e	f
49	a	b	c	d	e	f
50	a	b	c	d	e	f
51	a	b	c	d	e	f
52	a	b	c	d	e	f
53	a	b	c	d	e	f
54	a	b	c	d	e	f
55	a	b	c	d	e	f
56	a	b	c	d	e	f
57	a	b	c	d	e	f
58	a	b	c	d	e	f
59	a	b	c	d	e	f
60	a	b	c	d	e	f
61	a	b	c	d	e	f
62	a	b	c	d	e	f
63	a	b	c	d	e	f
64	a	b	c	d	e	f
65	a	b	c	d	e	f
66	a	b	c	d	e	f
67	a	b	c	d	e	f
68	a	b	c	d	e	f
69	a	b	c	d	e	f
70	a	b	c	d	e	f

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 003

Math 102

CODE 003

Final Exam

Summer (073)

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1.
$$\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$$

(a) $\frac{27}{4}$

(b) $\frac{31}{4}$

(c) $\frac{23}{4}$

(d) $\frac{9}{4}$

(e) $\frac{11}{4}$

2.
$$\int_0^{\pi} |\sin 2x| \, dx =$$

(a) 1

(b) 2

(c) -1

(d) 0

(e) $\frac{1}{2}$

3. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the y -axis, then the volume of the solid generated is equal to

(a) $\frac{\pi^2}{2}$

(b) $2\pi^2$

(c) $\frac{\pi^2}{4}$

(d) $4\pi^2$

(e) π^2

4. The series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 1}{2n^3 + n} \right)^n$

(a) Diverges by the limit comparison test

(b) Diverges by the Root test

(c) Diverges by the Ratio test

(d) Converges by the Root test

(e) is a convergent geometric series

5. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{3}$

(d) $\frac{2}{5}$

(e) $\frac{2}{3}$

6. $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} dx =$

(a) $x + \sin^{-1} x + c$

(b) $x + \sqrt{1-x^2} + c$

(c) $x - \frac{1}{2}\sqrt{1-x^2} + c$

(d) $x + \frac{3}{2}\sqrt{1-x^2} + c$

(e) $x - \sin^{-1} x + c$

7. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{5\pi}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{9\pi}{2}$

(d) π

(e) $\frac{7\pi}{2}$

8. The area of the surface obtained by rotating the curve $y = \sqrt{3 - x^2}$, $0 \leq x \leq 1$, about the x -axis is equal to

(a) $\frac{\pi\sqrt{3}}{6}$

(b) $\frac{4\sqrt{3}}{2}$

(c) $\frac{4\pi\sqrt{3}}{3}$

(d) $6\pi\sqrt{3}$

(e) $2\pi\sqrt{3}$

9. $\int \tan^{-1} \left(\frac{1}{x} \right) dx =$

(a) $x \tan^{-1} \left(\frac{1}{x} \right) - \ln(1 + x^2) + c$

(b) $\left(\frac{1}{x^2} \right) \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(c) $\frac{1}{2} x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(d) $x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1 + x^2} + c$

(e) $(x + 1) \tan^{-1} \left(\frac{1}{x} \right) + c$

10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5 6^n}{(n + 1)!}$

(a) is divergent by the ratio test

(b) is convergent by the integral test

(c) is divergent by the test for divergence

(d) is absolutely convergent

(e) is conditionally convergent

11. If $x > e$, then $\frac{d}{dx} \int_1^{\sqrt{\ln x}} e^{t^2} dt =$

(a) 0

(b) x

(c) $2x$

(d) $\frac{1}{2\sqrt{\ln x}}$

(e) $\frac{4}{\sqrt{\ln x}}$

12. $\int \frac{x^2 + x + 3}{(x - 1)(x^2 + 2x + 2)} dx =$

(a) $x + \ln|x - 1| + \tan^{-1}(x + 1) + c$

(b) $\ln|x - 1| - \tan^{-1}(x + 1) + c$

(c) $\ln|x - 1| + 2 \tan^{-1}(x + 1) + c$

(d) $2x + \ln|x - 1| - 3 \tan^{-1}(x + 1) + c$

(e) $\ln(x - 1)^2 - \tan^{-1}(x + 1) + c$

13. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4 + 5}}$
- (a) Converges by the integral test
 - (b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$
 - (c) Diverges by the test for divergence
 - (d) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$
 - (e) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$
14. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (2x - 3)^n}{n \cdot 4^n}$ are given by
- (a) $R = 2, I = [-4, 4]$
 - (b) $R = 2, I = \left(-\frac{1}{2}, \frac{7}{2}\right]$
 - (c) $R = 2, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$
 - (d) $R = 4, I = (-4, 4]$
 - (e) $R = 4, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

15. The value of the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \left(\cos \frac{i\pi}{2n} \right)^2$ on the interval $\left[0, \frac{\pi}{2} \right]$ is

(a) $1 + \frac{\pi}{2}$

(b) $\frac{\pi}{8}$

(c) $\frac{\pi}{2}$

(d) $-\frac{1}{4} + \frac{\pi}{8}$

(e) $1 + \frac{\pi}{8}$

16. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges conditionally

(b) Converges by the integral test

(c) Diverges

(d) Converges by the root test

(e) Converges absolutely

17. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}} dx =$

(a) $\sqrt{3}$

(b) $\frac{1}{3}$

(c) $\frac{7}{3}$

(d) $\frac{\sqrt{3}}{3}$

(e) $\frac{4}{3}$

18. The sequence $\left\{ (2n+1) \sin \frac{7}{n} \right\}$

(a) Converges to 14

(b) Converges to 0

(c) Diverges

(d) Converges to $\frac{2}{7}$

(e) Converges to $\frac{7}{2}$

19. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left(\frac{1}{5}, \infty\right)$

(b) $(0, \infty)$

(c) $\left[\frac{1}{5}, \infty\right)$

(d) $[1, \infty)$

(e) $(1, \infty)$

20. The improper integral $\int_{-\infty}^{\infty} e^{-|x|} dx$

(a) Converges to 2

(b) Converges to 1

(c) Diverges

(d) Converges to $\frac{1}{2}$

(e) Converges to 0

21. $\int_0^{\frac{\pi}{4}} \sqrt{\frac{1 + \sin x}{1 - \sin x}} dx =$

(a) $\ln(2\sqrt{2} - 1)$

(b) $\ln(1 + \sqrt{2})$

(c) $\ln(2 + \sqrt{2})$

(d) $\ln \sqrt{2}$

(e) $\ln(1 + 2\sqrt{2})$

22. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclaurin series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{14}{3}$

(b) $\frac{7}{3}$

(c) $\frac{29}{6}$

(d) $\frac{31}{6}$

(e) $\frac{5}{6}$

23. If $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$, then $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$

(a) π^2

(b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{4}$

(d) $\frac{\pi^2}{16}$

(e) π

24. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $0 < x < \pi$, we get

$$\int \frac{2 dx}{\sin x(1 + \cos x)} = A \ln\left(\tan \frac{x}{2}\right) + B \tan^2 \frac{x}{2} + c \text{ where } A + 2B =$$

(a) 4

(b) 1

(c) 0

(d) 2

(e) 3

25. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50 \cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

- (a) $100(e^2 + e^{-2})$
 - (b) $50(e^2 - e^{-2})^2$
 - (c) $100(e + e^{-1})$
 - (d) $50(e^2 - e^{-2})$
 - (e) $50(e - e^{-1})$
26. The improper integral $\int_0^1 \frac{1}{e^x - 1} dx$
- (a) Converges to $\ln(1 - e^{-1})$
 - (b) Converges to 1
 - (c) Diverges
 - (d) Converges to 0
 - (e) Converges to $\ln(e - 1)$

27. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 599

(b) 198

(c) 488

(d) 408

(e) 499

28. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} + \frac{2}{3}$

(b) $\frac{\pi}{4} + \frac{1}{3}$

(c) $\frac{\pi}{4} + \frac{4}{3}$

(d) $\frac{\pi}{4} - \frac{1}{3}$

(e) $\frac{\pi}{4} - \frac{2}{3}$

Name

ID

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69	a	b	c	d	e	f
70	a	b	c	d	e	f

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 004

Math 102

CODE 004

Final Exam

Summer (073)

Tuesday 26/8/2008

Net Time Allowed: 180 minutes

Name: _____

ID: _____ Sec: _____

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. $\int \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}} dx =$

(a) $x + \frac{3}{2}\sqrt{1-x^2} + c$

(b) $x + \sqrt{1-x^2} + c$

(c) $x + \sin^{-1} x + c$

(d) $x - \frac{1}{2}\sqrt{1-x^2} + c$

(e) $x - \sin^{-1} x + c$

2. $\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$

(a) $\frac{23}{4}$

(b) $\frac{11}{4}$

(c) $\frac{27}{4}$

(d) $\frac{31}{4}$

(e) $\frac{9}{4}$

3. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the y -axis, then the volume of the solid generated is equal to

(a) $\frac{\pi^2}{2}$

(b) $2\pi^2$

(c) $\frac{\pi^2}{4}$

(d) π^2

(e) $4\pi^2$

4. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{3}{4}$

(e) $\frac{2}{5}$

5. $\int_0^\pi |\sin 2x| dx =$

(a) 0

(b) -1

(c) 2

(d) $\frac{1}{2}$

(e) 1

6. The series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 1}{2n^3 + n} \right)^n$

(a) Diverges by the limit comparison test

(b) Diverges by the Ratio test

(c) is a convergent geometric series

(d) Converges by the Root test

(e) Diverges by the Root test

7. The improper integral $\int_{-\infty}^{\infty} e^{-|x|} dx$

(a) Converges to 2

(b) Converges to 0

(c) Converges to $\frac{1}{2}$

(d) Diverges

(e) Converges to 1

8. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}} dx =$

(a) $\sqrt{3}$

(b) $\frac{7}{3}$

(c) $\frac{\sqrt{3}}{3}$

(d) $\frac{1}{3}$

(e) $\frac{4}{3}$

9. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{9\pi}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{7\pi}{2}$

(d) π

(e) $\frac{5\pi}{2}$

10. If $x > e$, then $\frac{d}{dx} \int_1^{\sqrt{\ln x}} e^{t^2} dt =$

(a) $\frac{1}{2\sqrt{\ln x}}$

(b) x

(c) $\frac{4}{\sqrt{\ln x}}$

(d) $2x$

(e) 0

11. $\int \tan^{-1} \left(\frac{1}{x} \right) dx =$

(a) $x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

(b) $x \tan^{-1} \left(\frac{1}{x} \right) - \ln (1+x^2) + c$

(c) $\frac{1}{2} x \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

(d) $(x+1) \tan^{-1} \left(\frac{1}{x} \right) + c$

(e) $\left(\frac{1}{x^2} \right) \tan^{-1} \left(\frac{1}{x} \right) + \ln \sqrt{1+x^2} + c$

12. The value of the limit $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \left(\cos \frac{i\pi}{2n} \right)^2$ on the interval $\left[0, \frac{\pi}{2} \right]$ is

(a) $1 + \frac{\pi}{8}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{8}$

(d) $-\frac{1}{4} + \frac{\pi}{8}$

(e) $1 + \frac{\pi}{2}$

13. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^5 6^n}{(n+1)!}$
- (a) is divergent by the test for divergence
 - (b) is absolutely convergent
 - (c) is conditionally convergent
 - (d) is divergent by the ratio test
 - (e) is convergent by the integral test
14. The radius of convergence R and the interval of convergence I of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (2x-3)^n}{n \cdot 4^n}$ are given by
- (a) $R = 2, I = \left(-\frac{1}{2}, \frac{7}{2}\right]$
 - (b) $R = 4, I = (-4, 4]$
 - (c) $R = 2, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$
 - (d) $R = 2, I = [-4, 4]$
 - (e) $R = 4, I = \left[-\frac{1}{2}, \frac{7}{2}\right)$

15. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclarium series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{5}{6}$

(b) $\frac{7}{3}$

(c) $\frac{31}{6}$

(d) $\frac{29}{6}$

(e) $\frac{14}{3}$

16. $\int \frac{x^2 + x + 3}{(x - 1)(x^2 + 2x + 2)} dx =$

(a) $2x + \ln|x - 1| - 3 \tan^{-1}(x + 1) + c$

(b) $x + \ln|x - 1| + \tan^{-1}(x + 1) + c$

(c) $\ln(x - 1)^2 - \tan^{-1}(x + 1) + c$

(d) $\ln|x - 1| + 2 \tan^{-1}(x + 1) + c$

(e) $\ln|x - 1| - \tan^{-1}(x + 1) + c$

17. The area of the surface obtained by rotating the curve $y = \sqrt{3 - x^2}$, $0 \leq x \leq 1$, about the x -axis is equal to

(a) $\frac{4\pi\sqrt{3}}{3}$

(b) $2\pi\sqrt{3}$

(c) $6\pi\sqrt{3}$

(d) $\frac{\pi\sqrt{3}}{6}$

(e) $\frac{4\sqrt{3}}{2}$

18. The sequence $\left\{ (2n + 1) \sin \frac{7}{n} \right\}$

(a) Converges to $\frac{2}{7}$

(b) Diverges

(c) Converges to 0

(d) Converges to $\frac{7}{2}$

(e) Converges to 14

19. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

- (a) Converges conditionally
- (b) Converges absolutely
- (c) Converges by the integral test
- (d) Diverges
- (e) Converges by the root test

20. The series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4 + 5}}$

- (a) Diverges by the test for divergence
- (b) Diverges by the comparison test with $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$
- (c) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$
- (d) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$
- (e) Converges by the integral test

21. $\int_0^{\frac{\pi}{4}} \sqrt{\frac{1 + \sin x}{1 - \sin x}} dx =$

(a) $\ln(1 + 2\sqrt{2})$

(b) $\ln(2\sqrt{2} - 1)$

(c) $\ln(1 + \sqrt{2})$

(d) $\ln(2 + \sqrt{2})$

(e) $\ln \sqrt{2}$

22. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $(1, \infty)$

(b) $(0, \infty)$

(c) $\left(\frac{1}{5}, \infty\right)$

(d) $\left[\frac{1}{5}, \infty\right)$

(e) $[1, \infty)$

23. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, $0 < x < \pi$, we get

$$\int \frac{2 dx}{\sin x(1 + \cos x)} = A \ln\left(\tan \frac{x}{2}\right) + B \tan^2 \frac{x}{2} + c \text{ where } A+2B =$$

- (a) 1
- (b) 3
- (c) 4
- (d) 2
- (e) 0

24. The improper integral $\int_0^1 \frac{1}{e^x - 1} dx$

- (a) Converges to $\ln(1 - e^{-1})$
- (b) Converges to 0
- (c) Converges to 1
- (d) Diverges
- (e) Converges to $\ln(e - 1)$

25. If $\int_0^\pi x f(\sin x) dx = \frac{\pi}{2} \int_0^\pi f(\sin x) dx$, then $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$

(a) $\frac{\pi^2}{16}$

(b) π

(c) $\frac{\pi^2}{4}$

(d) $\frac{\pi}{2}$

(e) π^2

26. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50 \cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

(a) $100(e^2 + e^{-2})$

(b) $50(e^2 - e^{-2})$

(c) $100(e + e^{-1})$

(d) $50(e - e^{-1})$

(e) $50(e^2 - e^{-2})^2$

27. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} - \frac{1}{3}$

(b) $\frac{\pi}{4} - \frac{2}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4} + \frac{1}{3}$

(e) $\frac{\pi}{4} + \frac{4}{3}$

28. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 488

(b) 499

(c) 599

(d) 198

(e) 408

Name

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68	a	b	c	d	e	f
69	a	b	c	d	e	f
70	a	b	c	d	e	f

Q	MM	V1	V2	V3	V4
1	a	e	a	a	b
2	a	b	a	b	c
3	a	a	b	b	b
4	a	e	c	d	c
5	a	d	c	c	c
6	a	d	a	b	d
7	a	c	a	e	a
8	a	a	d	e	e
9	a	e	d	d	c
10	a	b	b	d	a
11	a	c	d	d	a
12	a	c	d	b	c
13	a	e	c	d	b
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24	a	b	b	d	d
25	a	d	d	d	c
26	a	b	a	c	b
27	a	b	a	e	b
28	a	a	c	e	b

Answer Counts

V	a	b	c	d	e
1	7	4	3	7	7
2	5	7	7	5	4
3	6	5	4	8	5
4	7	6	6	5	4