

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
DEPARTMENT OF MATHEMATICS AND STATISTICS

**Math 102 –Calculus II**

**Final Exam (072)**

**Code 001**

**Saturday: June 7, 2008**

**Time: 7:30 – 10:30 AM**

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**Student's Name:** .....

**ID #:** .....

**Section #:** .....

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**Important Instructions:**

1. All types of CALCULATORS, PAGERS, OR MOBILES ARE NOT ALLOWED to be with you during the examination.
2. Use an HB 2 pencil.
3. Use a good eraser. Do not use the eraser attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that bubbles match with the number that you write.
6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.
9. Check that the exam paper has **28** questions.

1. The volume of the solid generated by revolving the region enclosed by the curve  $y = \sin(x^2)$  and the  $x$ -axis over the interval  $[0, \sqrt{\pi}]$  about the  $y$ -axis is

- (a)  $\pi$
- (b)  $2\pi$
- (c)  $-2\pi$
- (d)  $\frac{\pi}{2}$
- (e)  $3\sqrt{\pi}$

2. The volume obtained by rotating the region bounded by  $y = x^4$ ,  $y = 1$  about  $y = 1$  equals

- (a)  $\frac{2}{9}\pi$
- (b)  $\frac{22}{45}\pi$
- (c)  $\frac{64}{45}\pi$
- (d)  $\frac{34}{45}\pi$
- (e)  $\frac{44}{45}\pi$

3. The area of the region enclosed by the graphs of  $y^2 = x$  and  $y = x - 2$  is equal to

(a)  $\int_{-1}^2 (2 + y - y^2) dy$

(b)  $\int_1^4 (x^2 - x + 2) dx$

(c)  $\int_1^4 (\sqrt{x} - x + 2) dx$

(d)  $\int_{-1}^2 (x^2 - x + 2) dx$

(e)  $\int_1^4 (2 + y - y^2) dy$

4.  $\int \frac{dx}{\sqrt{6x - x^2}} =$

(a)  $\sin^{-1} \frac{x - 3}{3} + c$

(b)  $2\sqrt{6x - x^2} + c$

(c)  $\ln(6x - x^2) + c$

(d)  $\sin^{-1} \frac{3 - x}{3} + c$

(e)  $3 \sin^{-1} \frac{x - 3}{3} + c$

5. If  $f'$  is continuous function,  $f(1) = 3$ , and  $\int_0^3 x f'(1+x^2) dx = 4$ , then  $f(10) =$

(a) 9

(b) 11

(c) 8

(d) 10

(e) 5

6.  $\int_0^{\frac{\pi^2}{4}} \cos \sqrt{x} dx =$

(a) 0

(b)  $2 - \pi$

(c)  $\pi - 2$

(d)  $\frac{\pi}{2} - 1$

(e)  $1 + \frac{\pi}{4}$

7. The graph of the function  $f(x)$  is given in the figure. Find

$$\int_0^7 f(x)dx$$

(a)  $5 + 4\pi$

(b)  $5 + 2\pi$

(c)  $6 - 4\pi$

(d)  $5 - 4\pi$

(e)  $5 - 2\pi$

8.  $\int_0^1 \frac{1}{1 + e^{-x}} dx =$

(a)  $\ln(1 + e)$

(b)  $\ln 2$

(c)  $e$

(d)  $\ln \frac{(e + 1)}{2}$

(e)  $\ln 3$

9.  $\int_4^8 (\sqrt{x} + \frac{1}{\sqrt{x}})^2 dx =$

(a)  $32 + \ln 2$

(b)  $32 + \ln 4$

(c)  $38 + \ln 2$

(d)  $38 + \ln 4$

(e)  $64 + \ln 2$

10.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \frac{1}{1 + (\frac{i}{n})^2} =$

(a)  $\frac{1}{4}$

(b)  $\infty$

(c)  $\frac{\pi}{4}$

(d)  $\pi$

(e)  $\frac{\pi}{2}$

11. The area of the surface of the solid obtained by rotating the curve  $y = \sqrt{1 + e^x}$ ,  $0 \leq x \leq 1$  about the  $x$ -axis is equal to

(a)  $\pi(e + 1)$

(b)  $\pi(2e + 1)$

(c)  $2\pi(e - 1)$

(d)  $\pi e$

(e)  $\pi(e - 1)$

12. The length of the curve  $y = \ln(\cos x)$ ,  $0 \leq x \leq \frac{\pi}{3}$  is

(a)  $\ln(2 + \sqrt{3})$

(b)  $\ln 3$

(c)  $\csc\left(\frac{\pi}{2}\right) - \csc\left(\frac{\pi}{3}\right)$

(d)  $\ln\left(\frac{3}{2}\right)$

(e)  $-\frac{1}{2}$

13. The series  $\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$
- (a) converges by integral test
  - (b) converges by the test for divergence
  - (c) diverges by comparison test with  $\sum_{n=2}^{\infty} \frac{1}{n}$
  - (d) diverges by the test for divergence
  - (e) converges by ratio test

14.  $\int \frac{dx}{x^3 - x} =$

- (a)  $\frac{1}{2} \ln |x^2 - 1| - \ln |x| + c$
- (b)  $\frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| - \ln |x| + c$
- (c)  $\frac{1}{2} \ln |x^2 - 1| + \ln |x| + c$
- (d)  $\ln |x^2 - 1| + \ln |x| + c$
- (e)  $\frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + \ln |x| + c$



15. The series  $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
- (a) is absolutely convergent
  - (b) diverges by the test for divergence
  - (c) converges by the ratio test
  - (d) is a convergent  $p$ -series
  - (e) diverges by the integral test
16. For the convergent alternating series  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$ , what is the smallest number of terms needed to guarantee that  $S_n$  is within  $1 \times 10^{-8}$  of the actual sum  $S$ ?
- (a) 100
  - (b) 99
  - (c) 1000
  - (d) 10
  - (e) 80

17. The integral  $\int \frac{2}{x + \sqrt[3]{x}} dx$  equals

(a)  $\ln|x + \sqrt[3]{x}| + c$

(b)  $\ln\left(\frac{2}{3} + x^2\right) + c$

(c)  $\ln(2 + e^{-x}) + c$

(d)  $2x + 3\ln(x^{2/3} + 1) + c$

(e)  $3\ln(x^{2/3} + 1) + c$

18.  $\int_0^{\frac{\pi}{2}} \cos^2 x \sin 2x dx =$

(a) 0

(b)  $\frac{1}{4}$

(c)  $\frac{1}{2}$

(d)  $\frac{3}{8}$

(e) -2

19.  $\int \frac{1}{x^2(1+x^2)} dx =$

(a)  $-\frac{1}{x} - \tan^{-1} x + c$

(b)  $-\frac{1}{x^2} + \frac{2x}{1+x^2} + c$

(c)  $\ln |x| - \tan^{-1} x + c$

(d)  $\frac{1}{x} + \ln |1+x^2| + c$

(e)  $-\frac{1}{x} - \frac{1}{(1+x^2)^2} + c$

20. The series  $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(a) is a convergent geometric series

(b) is a convergent p-series

(c) converges to  $\ln 2$

(d) is divergent

(e) converges to 1

21. If  $I = \int_{-1}^1 \sin(x^2) dx$ , then

(a)  $0 \leq I \leq 2$

(b)  $I = \infty$

(c)  $I = 0$

(d)  $I > 2$

(e)  $I \leq 0$

22.  $\int_0^{\infty} x e^{-x} dx =$

(a)  $\frac{1}{e} + 1$

(b)  $\infty$

(c)  $-1$

(d)  $-2$

(e)  $1$

23. Using the power series of  $\ln(1 - x)$ , the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{n3^n} \text{ is equal to}$$

(a)  $\ln 3$

(b)  $1$

(c)  $\ln \frac{3}{2}$

(d)  $\ln \frac{2}{3}$

(e)  $\ln 2$

24. The sequence  $\left\{ \left( 1 + \frac{2}{n} \right)^n \right\}_{n=1}^{\infty}$

(a) converges to  $\sqrt{e}$

(b) converges to  $e^2$

(c) converges to  $e$

(d) diverges

(e) converges to  $2$

25. The series  $\sum_{k=0}^{\infty} \frac{(-1)^k k!}{e^k}$  is

- (a) convergent by the root test
- (b) convergent to  $e^{10}$
- (c) convergent by the ratio test
- (d) divergent
- (e) convergent to  $e^3$

26. The series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4 \sqrt{n}}$

- (a) is absolutely convergent
- (b) is conditionally convergent
- (c) has the sum  $s = \frac{2}{9}$
- (d) is divergent
- (e) is absolutely divergent

27. The first 5 terms of the Taylor series of the function  $f(x) = x \ln x$  at  $x = 1$  are

(a)  $(x - 1) + \frac{(x - 1)^2}{2} - \frac{(x - 1)^3}{6} + \frac{(x - 1)^4}{12} - \frac{(x - 1)^5}{20}$

(b)  $(x - 1) + \frac{(x - 1)^2}{2} + \frac{(x - 1)^3}{6} + \frac{(x - 1)^4}{12} + \frac{(x - 1)^5}{20}$

(c)  $(x - 1) + \frac{(x - 1)^2}{2!} - \frac{(x - 1)^3}{3!} + \frac{(x - 1)^4}{4!} - \frac{(x - 1)^5}{5!}$

(d)  $(x - 1) - \frac{(x - 1)^2}{2} + \frac{(x - 1)^3}{6} - \frac{(x - 1)^4}{12} + \frac{(x - 1)^5}{20}$

(e)  $(x - 1) + \frac{(x - 1)^2}{2!} + \frac{(x - 1)^3}{3!} + \frac{(x - 1)^4}{4!} + \frac{(x - 1)^5}{5!}$

28. The interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{(x - 2)^n}{n^3 2^n}$

(a)  $0 < x < 3$

(b)  $0 \leq x < 4$

(c)  $0 \leq x \leq 4$

(d)  $-\infty < x < \infty$

(e)  $0 < x \leq 4$

Q	MM	V1	V2	V3	V4
1	a	b	b	c	e
2	a	c	e	c	a
3	a	a	a	b	d
4	a	a	d	e	b
5	a	b	b	b	b
6	a	c	d	a	b
7	a	e	d	c	e
8	a	d	b	b	a
9	a	a	d	b	e
10	a	c	e	e	c
11	a	a	e	a	b
12	a	a	e	a	c
13	a	c	e	c	a
14	a	a	e	a	c
15	a	e	d	e	a
16	a	b	a	a	c
17	a	e	e	a	e
18	a	c	b	a	b
19	a	a	e	c	d
20	a	e	e	d	a
21	a	a	c	c	c
22	a	e	b	e	d
23	a	c	c	b	d
24	a	b	e	a	a
25	a	d	d	e	e
26	a	a	a	a	c
27	a	a	b	b	b
28	a	c	e	e	e