

1. The function  $f(x) = \begin{cases} ax^2 + bx & \text{if } x \leq 1 \\ x + a^2 & \text{if } x > 1 \end{cases}$

is twice differentiable everywhere. Then  $a^2 + b^2 =$

- a) 1
- b) 0
- c)  $\frac{5}{4}$
- d) 2
- e) 5

2. If  $f(x) = (2x - 1)^{\frac{2}{3}}$ , then the equation of the vertical tangent to the graph of  $f$  is

- a)  $x = \frac{1}{2}$
- b)  $x = -\frac{1}{2}$
- c)  $x = \frac{2}{3}$
- d)  $x = -\frac{2}{3}$
- e)  $x = \frac{4}{3}$

3. The equations of the horizontal tangents to the curve  $y = x^3 - 3x - 2$  are
- a)  $y = 0$  and  $y = -4$
  - b)  $y = 1$  and  $y = -1$
  - c)  $x = 1$  and  $x = -1$
  - d)  $y = -4$  and  $y = 1$
  - e)  $y = 0$  and  $y = -1$
4. At how many real values of  $x$  does the curve  $y = x^6 - 3x^2 + x + 5$  have a tangent line parallel to the line  $y = x$ ?
- a) 3
  - b) 1
  - c) 2
  - d) 4
  - e) 5

5. If  $f(x) = xe^x$  and  $n$  is a positive integer, then  $f^{(n)}(1) =$

- a)  $(n + 1) e$
- b)  $n e$
- c)  $(n - 1) e$
- d)  $(n + 2) e$
- e)  $n e + 1$

6. If  $y = \frac{1 + \sin x}{1 + \cos x}$ , then  $\frac{dy}{dx} =$

a)  $\frac{1 + \sin x + \cos x}{(1 + \cos x)^2}$

b)  $\frac{\sin x + \cos x}{1 + \cos x}$

c)  $\frac{\sin x + \cos x}{(1 + \cos x)^2}$

d)  $\frac{1 + \sin x}{(1 + \cos x)^2}$

e)  $\frac{2}{1 + \cos x}$

7.  $\lim_{\theta \rightarrow 1} \frac{\sin(\theta - 1)}{\theta^2 + \theta - 2} =$

a)  $\frac{1}{3}$

b) 0

c)  $\frac{1}{2}$

d) 2

e) 1

8. If  $y = \sin(x^2)$  and  $x = \cos t$ , then  $\frac{dy}{dt} =$

- a)  $-\sin 2t \cos(\cos^2 t)$
- b)  $\sin 2t \cos(\cos^2 t)$
- c)  $-\sin t \cos(\cos^2 t)$
- d)  $-\sin 2t \cos^3 t$
- e)  $\sin 2t \cos^3 t$

9. Let  $f$  and  $g$  be differentiable functions and  $h(x) = f(x^2g(x))$ . If  $g(2) = -2$  and  $g'(2) = 2$ , then  $h'(2) =$

- a) 0
- b)  $-2$
- c) 2
- d) 3
- e)  $-3$

10. The equation of the tangent line to the curve given implicitly by

$$\sqrt{x+y} = y^2$$

at the point  $(0, 1)$  is

- a)  $3y - x = 3$
- b)  $2y + x = 1$
- c)  $3y + x = 3$
- d)  $2y - x = 2$
- e)  $2y + x = 3$

11. The equation of the normal line to the curve  $y = \tan^{-1}(\sqrt{x-1})$  at  $x = 2$  is

- a)  $y = -4x + 8 + \frac{\pi}{4}$
- b)  $y = \frac{1}{4}x - \frac{1}{2} + \frac{\pi}{4}$
- c)  $y = 4x - 8 + \frac{\pi}{4}$
- d)  $y = -\frac{1}{4}x + \frac{1}{2} + \frac{\pi}{4}$
- e)  $y = -4x + 8 - \frac{\pi}{4}$

12. If  $f(x) = (x^2 + 2x)^{50}$ , then  $f^{(100)}(1) =$

- a)  $100!$
- b)  $100$
- c)  $0$
- d)  $3(99!)$
- e)  $2(50!)$

13. The slope of the tangent line to the graph of  $y = (2x + 1)^{\sin 3x}$  at  $x = \frac{\pi}{6}$  is

- a)  $2$
- b)  $4\left(\frac{\pi}{3} + 1\right)$
- c)  $6$
- d)  $2\left(\frac{\pi}{3} + 1\right)$
- e)  $\frac{4}{\frac{\pi}{3} + 1}$

14. If  $y = \frac{(x+2)^2(2x-1)^3}{\sqrt{x+1}}$ , then  $y'(0) =$

- a) 22
- b)  $-\frac{11}{2}$
- c) 44
- d) 24
- e) -11

15. The position function of a particle moving along a line is

$$s(t) = \sin t + \cos t$$

where  $t$  is measured in seconds and  $s$  in meters. The total distance traveled by the particle in the interval  $[0, \pi]$  is

- a)  $2\sqrt{2}$  meters
- b) 2 meters
- c) 4 meters
- d)  $2\sqrt{2} + 2$  meters
- e)  $2\sqrt{2} - 2$  meters



16. The position function of a particle moving along a line is

$$s(t) = t^3 - 6t^2 + 9t \quad (0 \leq t \leq 5).$$

The time interval(s) where the particle is moving forward is (are)

- a) (0, 1) and (3, 5)
  - b) (0, 3)
  - c) (0, 3) and (4, 5)
  - d) (1, 3)
  - e) (0, 2) and (3, 5)
17. The two equal sides of an isosceles triangle have length  $4m$ . If the angle between them is increasing at a rate of  $0.06 \text{ rad/s}$ , then the rate at which the area of the triangle is changing when the angle between the sides of the triangle is  $\frac{\pi}{3}$  equals
- a)  $0.24 \text{ m}^2/\text{s}$
  - b)  $-0.24 \text{ m}^2/\text{s}$
  - c)  $2.4 \text{ m}^2/\text{s}$
  - d)  $-2.4 \text{ m}^2/\text{s}$
  - e)  $0.024 \text{ m}^2/\text{s}$

18. If a snow ball melts so that its surface area decreases at a rate of  $1 \text{ cm}^2/\text{min}$ , then the rate at which the diameter changes, when the diameter is  $10 \text{ cm}$  equals

Hint: Surface area of a sphere =  $4\pi r^2$

- a)  $\frac{-1}{20\pi} \text{ cm/min}$
- b)  $\frac{1}{20\pi} \text{ cm/min}$
- c)  $\frac{-1}{40\pi} \text{ cm/min}$
- d)  $\frac{1}{40\pi} \text{ cm/min}$
- e)  $\frac{-1}{10\pi} \text{ cm/min}$

19. The equation of the tangent line to the graph of  $y = \ln x$  and passes through the origin is

- a)  $e y = x$
- b)  $y = e x$
- c)  $y = \frac{1}{e}(x - 1)$
- d)  $y = \frac{1}{e}(x + 1)$
- e)  $y = 2 e x$