King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

MATH 101 EXAM I
Summer Term (083)

Time allowed: 120 Minutes

Name: _____ ID#:_____

Instructor: _____ Section:_____ Serial#:_____

- **Show All Your WORK**
- **WRITE Clear Steps**
- **Calculator and Mobiles are not allowed**

Q#	Marks	Maximum Marks
1		8
2		10
3		24
4		10
5		8
6		8
7		7
8		7
9		8
10		10
Total		100

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1. **(8 - points)** Sketch the graph of an example of a function f that satisfies the following conditions:

   $\lim_{x\to-\infty} f(x) = 3$; $\lim_{x\to\infty} f(x) = 1$; $\lim_{x\to1^-} f(x) = -\infty$;

   $f'(-2) = 0$; $\lim_{x\to1^+} f(x) = 2$; f has a removable discontinuity at $x = -1$.

2. **(10 - points)** Use the Squeeze Theorem to show that

$$\lim_{x\to0^+}\left(\sqrt{x}\ e^{\sin\left(\frac{\pi}{\sqrt{x}}\right)}+1\right)=1$$

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3. **(24 points: 6 points each)** Evaluate the limit, if it exists

(3a) $\lim\limits_{x\to 1} \dfrac{x^3-1}{\sqrt{2x+2}-2}$

(3b) $\lim\limits_{x\to 1^-} \dfrac{x^2-|x-1|-1}{|x-1|}$

(3c) $\lim\limits_{x\to \frac{1}{2}} (x-[|2x|])$, where $[|\ |]$ denotes the greatest integer function.

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(3d) $\lim_{x\to\infty} \ln\left(\dfrac{e^{x+2}-8}{e^x+16}\right)$

4. **(10 - points)** The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $S(t) = \dfrac{3t-1}{t+2}$ where t is measured in seconds. Use limits to find the instantaneous velocity at $t = 3$.

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5. **(8 - points)** Use the Intermediate Value Theorem to show that the graphs of the functions $f(x) = \sqrt{x}$ and $g(x) = \cos x$ intersect on the interval $\left[0, \frac{\pi}{2}\right]$.

6. Given that $f(x) = (x-1)^{\frac{2}{3}}$ and $f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}}$.

   (6a) **(3 - points)** Use limits to find, if any, the equation of the vertical tangent to the graph of f.

   (6b) **(5 - points)** Find the equation of the normal line to the graph of f at $x = 9$.

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7. **(7 - points)** Determine the intervals on which the function $f(x) = \dfrac{\ln(x) + \tan^{-1}(3x)}{x^2 - 4}$ is continuous.

8. **(7 - points)** Use limits to determine whether or not the following function is continuous at $x = 2$

$$f(x) = \begin{cases} \dfrac{10}{3x-1}, & \text{if } x < 2 \\ \sqrt{3x-2}, & \text{if } x \geq 2 \end{cases}$$

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9. **(8 - points)** Given that $\lim_{x\to 2}\left(3x-\frac{2}{5}\right)=\frac{28}{5}$ and $\epsilon = 0.009$. Find the largest possible value of δ that satisfies the conditions given in the $\epsilon-\delta$ definition of a limit.

10. **(10 - points)** Use limits to find all vertical and horizontal asymptotes of the graph of

$$f(x)=\frac{6x}{\sqrt{2x^2-8}}$$