King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 073
Wednesday 27/8/2008
Net Time Allowed: 180 minutes

MASTER VERSION

1. The value of the limit $\lim_{x \to 0^-} \dfrac{1}{1 + e^{1/x}}$ is equal to

(a) 1

(b) 0

(c) $-\infty$

(d) $+\infty$

(e) $\dfrac{1}{2}$

2. $\lim_{t \to \pi/4} \ln |1 - \sin(2t)| =$

(a) $-\infty$

(b) $+\infty$

(c) $\ln 2$

(d) 0

(e) $-\ln 2$

3. Using the ϵ, δ definition of limit to prove that $\lim_{x \to -\frac{1}{2}} (2x+4) = 3$, we can choose

(a) $\delta = \frac{\epsilon}{2}$

(b) $\delta = \epsilon$

(c) $\delta = \frac{2\epsilon}{3}$

(d) $\delta = \frac{\epsilon + 1}{2}$

(e) $\delta = \frac{4\epsilon}{5}$

4. The function $f(x) = \frac{\sqrt{2-x}}{x^2+1}$ is continuous on

(a) $(-\infty, 2]$

(b) $(-\infty, -1) \cup (-1, 1) \cup (1, 2]$

(c) $(-\infty, 4]$

(d) $[2, +\infty)$

(e) $(-\infty, +\infty)$

5. Which one of the following statements is **TRUE** about the function

$$f(x) = \begin{cases} 12 - x^2 & \text{if} \quad x \leq 3 \\ 2x + 3 & \text{if} \quad x > 3 \end{cases}$$

(a) f is not differentiable at $x = 3$

(b) f is continuous at $x = 3$

(c) f is differentiable at $x = 3$

(d) $\lim_{x \to 3^-} f(x) = 9$

(e) $f'(4) = -8$

6. The slope of the tangent line to the curve $y = |2x - 5|$ at $x = 0$

(a) is equal to -2

(b) is equal to 2

(c) is equal to 0

(d) is equal to 3

(e) Does not exist

7. An equation for the tangent line to the curve $y = xe^{-\frac{1}{x}}$ at $x = -1$ is given by

(a) $y = -e$

(b) $y = 0$

(c) $y = 2ex + e$

(d) $y = -ex - 2e$

(e) $y = ex + 2e$

8. If $H(x) = f(g(x))$, $g(2) = 4$, $g'(2) = 3$, $f'(4) = -1$, and $f'(3) = 5$, then $H'(2) =$

(a) -3

(b) 15

(c) 3

(d) -5

(e) -4

9. If $y = \left(\dfrac{x+1}{x-1} \right)^5$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{-10(x+1)^4}{(x-1)^6}$

(b) $5\left(\dfrac{x+1}{x-1}\right)^4$

(c) $\dfrac{10(x+1)^4}{x-1)^6}$

(d) $\dfrac{-5(x+1)^3}{(x-1)^6}$

(e) $\dfrac{-2(x+1)^4}{(x-1)^6}$

10. If $f(x) = \tan^{-1}(x^2)$, then $f'''(1) =$

(a) -2

(b) -1

(c) 0

(d) $\dfrac{3}{8}$

(e) $\dfrac{1}{4}$

11. If $\dfrac{y}{1+x} = \dfrac{x}{1+y}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{2x+1}{2y+1}$

(b) $\dfrac{(1+y)^2}{(1+x)^2}$

(c) $\dfrac{x+1}{(1+y)^2}$

(d) $\dfrac{x-1}{2y+x}$

(e) $\dfrac{x+1}{2y-1}$

12. The linear approximation of $f(x) = \sqrt[3]{1-9x}$ at 0 is

(a) $\sqrt[3]{1-9x} \approx 1-3x$

(b) $\sqrt[3]{1-9x} \approx 1-9x$

(c) $\sqrt[3]{1-9x} \approx 1-\dfrac{1}{3}x$

(d) $\sqrt[3]{1-9x} \approx -3x$

(e) $\sqrt[3]{1-9x} \approx 1+x$

13. $\lim_{x \to +\infty} x^2 e^{-3x} =$

(a) 0

(b) $\frac{2}{9}$

(c) $+\infty$

(d) 1

(e) $\frac{2}{3}$

14. The critical numbers of the function $f(x) = 2\cos x + \sin^2 x$ are

(a) $n\pi$, where n is an integer

(b) $2n\pi$, where n is an integer

(c) $\frac{(2n+1)}{2}\pi$, where n is an integer

(d) $\frac{n\pi}{2}$, where n is an integer

(e) $\frac{2n\pi}{3}$, where n is an integer

15. Which one of the following statements is **TRUE** about the function $f(x) = \dfrac{x^2 + x - 1}{x + 1}$

(a) The line $y = x$ is a slant (oblique) asymptote for f.

(b) The line $y = x + 1$ is a slant (oblique) asymptote for f.

(c) The line $y = -1$ is a vertical asymptote for f.

(d) The line $y = 1$ is a horizontal asymptote for f.

(e) f has no asymptotes

16. If the area of an equilateral triangle is increasing at a rate of $25\, m^2/h$, then the rate at which the sides of the triangle are increasing when the length of each side is $10\, m$ is

(a) $\dfrac{5\sqrt{3}}{3}\ m/h$

(b) $5\sqrt{3}\ m/h$

(c) $2\sqrt{3}\ m/h$

(d) $\dfrac{4\sqrt{3}}{3}\ m/h$

(e) $3\sqrt{3}\ m/h$

17. The number(s) c satisfying the conclusion of Rolle's Theorem for $f(x) = \sqrt{x(1-x)}$ on the interval $[0,1]$ is (are)

(a) $c = \frac{1}{2}$ only

(b) $c = -\frac{1}{2}$ and $c = \frac{1}{2}$

(c) $c = \frac{1}{4}$ only

(d) $c = 0$ only

(e) $c = 0$ and $c = \frac{1}{2}$

18. Newton's Method is used to find a root of the equation

$$2 \sin h\, x = \sqrt{e^x + 2}.$$

If the first approximation is $x_1 = \ln 2$, then the second approximation x_2 is equal to

(a) $\dfrac{1}{4} + \ln 2$

(b) $-\dfrac{1}{2} + \ln 2$

(c) $\ln \sqrt{2}$

(d) $\dfrac{3}{4} + \ln 2$

(e) $-\dfrac{3}{4} + \ln 2$

19. If $f(x) = \dfrac{(ax^2+1)^4}{(bx+1)^3(cx-1)^5}$, where $a, b,$ and c are constants, then $f'(0) =$

(a) $3b - 5c$

(b) $3a + 5c$

(c) $-8a + 3b - 5c$

(d) 0

(e) $a - 3b$

20. If the side of a square is measured to be 10 feet, with a possible error in measurement of 0.1 feet, then the estimated percentage error in the area of the square is

(a) 2 %

(b) 0.5 %

(c) 4 %

(d) 0.02 %

(e) 0.0002 %

21. The function $f(x) = x\sqrt{x+3}$

(a) has a local minimum at $x = -2$

(b) has a local maximum at $x = -2$

(c) is increasing on the interval $(-3, 2)$

(d) is decreasing on the interval $(-5/2, +\infty)$

(e) has a local minimum at $x = -1$

22. If the polynomial $f(x) = x^3 - kx^2 + kx + k$ has an inflection point at $x = 5$, then $k =$

(a) 15

(b) 3

(c) 10

(d) 6

(e) 7

23. If $f'(x) = \sqrt{x} \cdot (6 + 5x)$ and $f(1) = 10$, then $f(4) =$

(a) 100

(b) 96

(c) 68

(d) 36

(e) 52

24. If $y = (\tan x)^{\tan x}$, then $\dfrac{dy}{dx} =$

(a) $y \cdot (1 + \ln(\tan x)) \cdot \sec^2 x$

(b) $y \cdot (1 + \sec^2 x \ln(\tan x))$

(c) $y \cdot (\tan x + \ln(\tan x)) \cdot \sec^2 x$

(d) $y \cdot (\cot x + \ln(\tan x)) \cdot \sec^2 x$

(e) $y \cdot (\tan x + \sec^2 x)$

25. The value of the limit $\lim_{x\to 0^+} (1+5x)^{3\csc(2x)}$ is equal to

(a) $e^{15/2}$

(b) e^{30}

(c) $e^{6/5}$

(d) $e^{10/3}$

(e) 1

26. The two numbers x and y such that $x+y=120$ and the product x^3y^2 is a maximum are

(a) $x=72$ and $y=48$

(b) $x=84$ and $y=36$

(c) $x=27$ and $y=93$

(d) $x=64$ and $y=56$

(e) $x=57$ and $y=63$

27. The absolute minimum value of $f(x) = x - 2\ln x$ on the interval $[1, e^3]$ is

(a) $2 - \ln 4$

(b) 1

(c) $e^3 - 6$

(d) $2 - \ln 2$

(e) $e^3 - 2$

28. Which one of the following statements is **FALSE** about the graph of the function

$$f(x) = e^{2x-x^2}$$

(a) The absolute maximum value of f is 1

(b) The domain of f is $(-\infty, +\infty)$

(c) The line $y = 0$ is a horizontal asymptote for f

(d) The graph of f has an inflection point at $x = \dfrac{2+\sqrt{2}}{2}$

(e) f is increasing on the interval $(-\infty, 1)$