

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

CODE 001

**Math 101
Final Exam
061**

CODE 001

**Wednesday 24/1/2007
Net Time Allowed: 150 minutes**

Name: _____

ID: _____ Sec: _____

Check that this exam has 25 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y = \ln\left(\frac{x}{4}\right)$ and the value of x decreases from 4 to 3.9, then the corresponding change in y is approximated by the differential dy , which equals

(a) $-\frac{1}{100}$

(b) $-\frac{1}{40}$

(c) $\frac{1}{100}$

(d) $\frac{1}{40}$

(e) $-\ln\left(\frac{3.9}{4}\right)$

2. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$, then $\left(\frac{f}{f-g}\right)'(3)$ is equal to

(a) 16

(b) 32

(c) 8

(d) 44

(e) 12

3. The function $f(x) = x^4(x - 1)^3$ has
- (a) one local maximum and one local minimum
 - (b) two local maxima and one local minimum
 - (c) three local maxima and four local minima
 - (d) two local maxima and two local minima
 - (e) one local maximum and two local minima
4. The slope of the tangent line to the graph of $x^2y^2 + x \sin y = 4$ at the point $\left(\frac{2}{\pi}, \pi\right)$ is equal to
- (a) $-\frac{2}{3}\pi^2$
 - (b) $\frac{1}{2}\pi^2$
 - (c) $-\frac{3}{2}\pi$
 - (d) $\frac{3}{2}\pi$
 - (e) $-\frac{1}{2}\pi^2$

5. The number of the **different equations** of the normal lines of slope 1 to the graph of $y = \frac{1}{x+1}$ is

(a) 1

(b) 2

(c) 4

(d) 3

(e) 0

6. The graph of function $f(x) = x + 2 \sin x$ for $0 \leq x < 2\pi$, is concave up on the interval I and has inflection point $P(x, y)$ where

(a) $I = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; $P(\pi, \pi)$

(b) $I = (\pi, 2\pi)$; $P(\pi, \pi)$

(c) $I = (\pi, 2\pi)$; $P\left(\frac{\pi}{2}, \frac{\pi}{2} + 2\right)$

(d) $I = (0, \pi)$; $P(0, 0)$

(e) $I = (\pi, 2\pi)$; $P(0, 0)$

7. The value(s) of constant c making the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

- (a) 4 and -4
 - (b) -2
 - (c) 20
 - (d) -2 and 2
 - (e) 4
8. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, then the rate at which **the diameter** decreases, when the diameter is 10 cm, is

$$[\text{Hint: } (S = 4\pi r^2) \text{ and } (V = \frac{4}{3}\pi r^3)]$$

- (a) $20\pi \text{ cm/min}$
- (b) $\frac{1}{20\pi} \text{ cm/min}$
- (c) $\frac{4}{30\pi} \text{ cm/min}$
- (d) $10\pi \text{ cm/min}$
- (e) $\frac{1}{10\pi} \text{ cm/min}$

9. If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $-\frac{3}{2}$

(b) $-\frac{5}{2}$

(c) $\frac{1}{2}$

(d) $\frac{3}{2}$

(e) $-\frac{1}{2}$

10. Let g be a twice differentiable function. If $f(x) = g(\sqrt{x})$, then $f''(x)$ is equal to

(a) $\frac{1}{4x}g''(\sqrt{x}) - \frac{1}{4\sqrt{x^3}}g'(\sqrt{x})$

(b) $\frac{1}{4x}g''(\sqrt{x})$

(c) $\frac{-1}{4\sqrt{x^3}}g''(\sqrt{x})$

(d) $\frac{1}{2\sqrt{x}}g''(\sqrt{x})$

(e) $\frac{1}{2\sqrt{x}}g''(\sqrt{x}) - \frac{1}{4x}$

11. If $y = (\ln x)^{\ln x}$, then

(a) $\frac{dy}{dx} = (\ln x)^{\ln x - 1}$

(b) $\frac{dy}{dx} = \frac{\ln(\ln x + 1)}{x}$

(c) $\frac{dy}{dx} = \left(\frac{\ln(\ln x) + 1}{x} \right) (\ln x)^{\ln x}$

(d) $\frac{dy}{dx} = \frac{1}{x} \ln(\ln x)$

(e) $\frac{dy}{dx} = \frac{1}{x} (\ln x)^{\ln x - 1}$

12. $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$ is equal to

(a) 3

(b) 1

(c) $\frac{3}{2}$

(d) -1

(e) $-\frac{3}{2}$

13. Given $f(x) = 4x - 5$ and $\epsilon > 0$. The largest possible value of δ , such that $|f(x) - 7| < \epsilon$ whenever $|x - 3| < \delta$, is given by

(a) $\frac{\epsilon}{2}$

(b) $\frac{\epsilon}{5}$

(c) ϵ

(d) $\frac{\epsilon}{4}$

(e) $\frac{\epsilon}{3}$

14. Which one of the following statements is **TRUE** about the graph of the function

$$y = \frac{x}{x^2 - x - 2}?$$

- (a) It has a y -intercept and one vertical asymptote
- (b) It has two vertical asymptotes and no horizontal asymptote
- (c) It has two x -intercepts and one asymptote at $x = 0$
- (d) It passes through the origin and has asymptotes at $x = 2$ and $x = -1$
- (e) It passes through origin and has asymptotes at $x = 1$ and $x = \frac{-1}{2}$

15. If $f'(x) = \frac{4}{\sqrt{1-x^2}}$ and $f\left(\frac{1}{2}\right) = 1$, then $1 - f\left(-\frac{1}{2}\right) =$

(a) 2

(b) 0

(c) $\frac{3\pi}{4}$

(d) $-\frac{3\pi}{4}$

(e) $\frac{4\pi}{3}$

16. The picture shows the graphs of f , f' and f'' . Identify each curve.

(a) $f = C$, $f' = B$, $f'' = A$

(b) $f = B$, $f' = A$, $f'' = C$

(c) $f = B$, $f' = C$, $f'' = A$

(d) $f = C$, $f' = A$, $f'' = B$

(e) $f = A$, $f' = B$, $f'' = C$

17. $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)}$ is equal to

- (a) $\ln 2$
- (b) $1 + \ln 2$
- (c) 0
- (d) 2
- (e) 1

18. Starting with $x_1 = 1$, the next approximation x_2 to a root of $\tan^{-1} x = 1 - x$ by Newton's method is

[Use $\pi = \frac{22}{7}$]

- (a) $\frac{13}{21}$
- (b) $\frac{8}{21}$
- (c) $\frac{19}{21}$
- (d) $\frac{10}{21}$
- (e) $\frac{14}{21}$

19. Which of the following statements is **TRUE** about $f(x) = x^{2/3}$?

- (a) f has no vertical tangent line
- (b) f is differentiable on $(-\infty, \infty)$
- (c) f has a vertical tangent line at $(0, 0)$
- (d) f has a horizontal tangent line
- (e) f has a vertical tangent line at $x = 1$

20. If M and m are, respectively, the absolute maximum and minimum of $f(x) = xe^{-x}$ on the interval $[0, 2]$, then $e^2M - em =$

- (a) e
- (b) $\frac{e+1}{e}$
- (c) $-e$
- (d) $2e$
- (e) 2

21. The values for f, g, f', g' are given by the table below:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	9	8	5
4	-3	3	2	-6

Then $\frac{d}{dx}(g(x + f(x)))$ at $x = 3$ is

- (a) 27
- (b) -48
- (c) 5
- (d) -54
- (e) -6
22. The x-coordinate of the point at which the tangent line to the curve $y = \cosh(2x)$ has slope 2, is equal to

- (a) $\frac{2}{3} \ln(1 + \sqrt{2})$
- (b) $2 \ln(1 + \sqrt{2})$
- (c) $3 \ln(1 + \sqrt{2})$
- (d) $\frac{3}{2} \ln(1 + \sqrt{2})$
- (e) $\frac{1}{2} \ln(1 + \sqrt{2})$

23. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then which of the following statements is **TRUE**?

[Hint: Apply the Mean Value Theorem]

- (a) $f(4) = 15$
 - (b) $f(4) < 16$
 - (c) $f(4) = 0$
 - (d) $f(4) \geq 16$
 - (e) $f(4) \leq 10$
24. The **height** of the right circular cylinder of largest volume which is inscribed in a sphere of radius 9 cm is

- (a) $\frac{5\sqrt{3}}{2}$ cm
- (b) $\frac{9\sqrt{3}}{2}$ cm
- (c) $6\sqrt{3}$ cm
- (d) $4\sqrt{3}$ cm
- (e) $9\sqrt{3}$ cm

25. Given $f(x) = \frac{x-1}{x^2}$, $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{2(x-3)}{x^4}$, Then which one of the following statements is **FALSE** about the graph of the function $f(x)$.

[Hint: Sketch]

- (a) the graph has one inflection point
- (b) the graph intersects its horizontal asymptote
- (c) the graph has one local maximum
- (d) the graph is concave up on $(3, \infty)$
- (e) the graph is concave up on $(2, 3)$

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CODE 002

**Math 101
Final Exam
061**

CODE 002

**Wednesday 24/1/2007
Net Time Allowed: 150 minutes**

Name: _____

ID: _____ Sec: _____

Check that this exam has 25 questions.

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8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $-\frac{1}{2}$

(b) $-\frac{5}{2}$

(c) $\frac{3}{2}$

(d) $\frac{1}{2}$

(e) $-\frac{3}{2}$

2. The function $f(x) = x^4(x - 1)^3$ has

(a) two local maxima and two local minima

(b) two local maxima and one local minimum

(c) one local maximum and one local minimum

(d) three local maxima and four local minima

(e) one local maximum and two local minima

3. Which one of the following statements is **TRUE** about the graph of the function

$$y = \frac{x}{x^2 - x - 2}?$$

- (a) It passes through origin and has asymptotes at $x = 1$ and $x = \frac{-1}{2}$
- (b) It has two x -intercepts and one asymptote at $x = 0$
- (c) It has two vertical asymptotes and no horizontal asymptote
- (d) It passes through the origin and has asymptotes at $x = 2$ and $x = -1$
- (e) It has a y -intercept and one vertical asymptote
4. Let g be a twice differentiable function. If $f(x) = g(\sqrt{x})$, then $f''(x)$ is equal to

- (a) $\frac{1}{4x}g''(\sqrt{x})$
- (b) $\frac{1}{2\sqrt{x}}g''(\sqrt{x})$
- (c) $\frac{1}{4x}g''(\sqrt{x}) - \frac{1}{4\sqrt{x^3}}g'(\sqrt{x})$
- (d) $\frac{-1}{4\sqrt{x^3}}g''(\sqrt{x})$
- (e) $\frac{1}{2\sqrt{x}}g''(\sqrt{x}) - \frac{1}{4x}$

5. If $y = (\ln x)^{\ln x}$, then

(a) $\frac{dy}{dx} = \frac{1}{x} \ln(\ln x)$

(b) $\frac{dy}{dx} = \frac{\ln(\ln x + 1)}{x}$

(c) $\frac{dy}{dx} = \left(\frac{\ln(\ln x) + 1}{x} \right) (\ln x)^{\ln x}$

(d) $\frac{dy}{dx} = (\ln x)^{\ln x - 1}$

(e) $\frac{dy}{dx} = \frac{1}{x} (\ln x)^{\ln x - 1}$

6. The number of the **different equations** of the normal lines of slope 1 to the graph of $y = \frac{1}{x+1}$ is

(a) 3

(b) 1

(c) 4

(d) 2

(e) 0

7. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, then the rate at which **the diameter** decreases, when the diameter is 10 cm, is

$$[\text{Hint: } (S = 4\pi r^2) \text{ and } (V = \frac{4}{3}\pi r^3)]$$

- (a) $\frac{4}{30\pi} \text{ cm/min}$
- (b) $\frac{1}{20\pi} \text{ cm/min}$
- (c) $\frac{1}{10\pi} \text{ cm/min}$
- (d) $10\pi \text{ cm/min}$
- (e) $20\pi \text{ cm/min}$
8. If $f'(x) = \frac{4}{\sqrt{1-x^2}}$ and $f\left(\frac{1}{2}\right) = 1$, then $1 - f\left(-\frac{1}{2}\right) =$

- (a) 2
- (b) $\frac{3\pi}{4}$
- (c) $-\frac{3\pi}{4}$
- (d) 0
- (e) $\frac{4\pi}{3}$

9. Given $f(x) = 4x - 5$ and $\epsilon > 0$. The largest possible value of δ , such that $|f(x) - 7| < \epsilon$ whenever $|x - 3| < \delta$, is given by

(a) $\frac{\epsilon}{5}$

(b) ϵ

(c) $\frac{\epsilon}{2}$

(d) $\frac{\epsilon}{4}$

(e) $\frac{\epsilon}{3}$

10. $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$ is equal to

(a) -1

(b) 3

(c) 1

(d) $\frac{3}{2}$

(e) $-\frac{3}{2}$

11. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$, then $\left(\frac{f}{f-g}\right)'(3)$ is equal to
- (a) 16
 - (b) 8
 - (c) 44
 - (d) 12
 - (e) 32
12. If $y = \ln\left(\frac{x}{4}\right)$ and the value of x decreases from 4 to 3.9, then the corresponding change in y is approximated by the differential dy , which equals
- (a) $\frac{1}{40}$
 - (b) $-\frac{1}{100}$
 - (c) $-\frac{1}{40}$
 - (d) $\frac{1}{100}$
 - (e) $-\ln\left(\frac{3.9}{4}\right)$

13. The value(s) of constant c making the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

- (a) -2
 - (b) 4 and -4
 - (c) 4
 - (d) -2 and 2
 - (e) 20
14. The slope of the tangent line to the graph of $x^2y^2 + x \sin y = 4$ at the point $\left(\frac{2}{\pi}, \pi\right)$ is equal to

- (a) $-\frac{3}{2}\pi$
- (b) $\frac{1}{2}\pi^2$
- (c) $\frac{3}{2}\pi$
- (d) $-\frac{2}{3}\pi^2$
- (e) $-\frac{1}{2}\pi^2$

15. The graph of function $f(x) = x + 2 \sin x$ for $0 \leq x < 2\pi$, is concave up on the interval I and has inflection point $P(x, y)$ where

(a) $I = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; $P(\pi, \pi)$

(b) $I = (\pi, 2\pi)$; $P\left(\frac{\pi}{2}, \frac{\pi}{2} + 2\right)$

(c) $I = (\pi, 2\pi)$; $P(\pi, \pi)$

(d) $I = (\pi, 2\pi)$; $P(0, 0)$

(e) $I = (0, \pi)$; $P(0, 0)$

16. The picture shows the graphs of f , f' and f'' . Identify each curve.

(a) $f = B$, $f' = C$, $f'' = A$

(b) $f = B$, $f' = A$, $f'' = C$

(c) $f = A$, $f' = B$, $f'' = C$

(d) $f = C$, $f' = A$, $f'' = B$

(e) $f = C$, $f' = B$, $f'' = A$

17. The x-coordinate of the point at which the tangent line to the curve $y = \cosh(2x)$ has slope 2, is equal to

(a) $\frac{3}{2} \ln(1 + \sqrt{2})$

(b) $\frac{2}{3} \ln(1 + \sqrt{2})$

(c) $\frac{1}{2} \ln(1 + \sqrt{2})$

(d) $2 \ln(1 + \sqrt{2})$

(e) $3 \ln(1 + \sqrt{2})$

18. If M and m are, respectively, the absolute maximum and minimum of $f(x) = xe^{-x}$ on the interval $[0, 2]$, then $e^2M - em =$

(a) $-e$

(b) $\frac{e+1}{e}$

(c) e

(d) $2e$

(e) 2

19. The values for f, g, f', g' are given by the table below:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	9	8	5
4	-3	3	2	-6

Then $\frac{d}{dx}(g(x + f(x)))$ at $x = 3$ is

- (a) -6
 - (b) -54
 - (c) 5
 - (d) 27
 - (e) -48
20. Given $f(x) = \frac{x-1}{x^2}$, $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{2(x-3)}{x^4}$, Then which one of the following statements is **FALSE** about the graph of the function $f(x)$.

[Hint: Sketch]

- (a) the graph is concave up on $(2, 3)$
- (b) the graph is concave up on $(3, \infty)$
- (c) the graph has one inflection point
- (d) the graph has one local maximum
- (e) the graph intersects its horizontal asymptote

21. The **height** of the right circular cylinder of largest volume which is inscribed in a sphere of radius 9 cm is

(a) $6\sqrt{3}$ cm

(b) $9\sqrt{3}$ cm

(c) $\frac{9\sqrt{3}}{2}$ cm

(d) $\frac{5\sqrt{3}}{2}$ cm

(e) $4\sqrt{3}$ cm

22. Starting with $x_1 = 1$, the next approximation x_2 to a root of $\tan^{-1} x = 1 - x$ by Newton's method is

[Use $\pi = \frac{22}{7}$]

(a) $\frac{19}{21}$

(b) $\frac{14}{21}$

(c) $\frac{8}{21}$

(d) $\frac{13}{21}$

(e) $\frac{10}{21}$

23. Which of the following statements is **TRUE** about $f(x) = x^{2/3}$?

(a) f has a vertical tangent line at $x = 1$

(b) f has no vertical tangent line

(c) f has a horizontal tangent line

(d) f has a vertical tangent line at $(0, 0)$

(e) f is differentiable on $(-\infty, \infty)$

24. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then which of the following statements is **TRUE**?

[Hint: Apply the Mean Value Theorem]

(a) $f(4) < 16$

(b) $f(4) = 0$

(c) $f(4) \geq 16$

(d) $f(4) = 15$

(e) $f(4) \leq 10$

25. $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)}$ is equal to

- (a) 0
- (b) 2
- (c) 1
- (d) $\ln 2$
- (e) $1 + \ln 2$

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

CODE 003

**Math 101
Final Exam
061**

CODE 003

**Wednesday 24/1/2007
Net Time Allowed: 150 minutes**

Name: _____

ID: _____ Sec: _____

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1. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$, then $\left(\frac{f}{f-g}\right)'(3)$ is equal to

(a) 16

(b) 12

(c) 8

(d) 44

(e) 32

2. If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $-\frac{3}{2}$

(b) $\frac{3}{2}$

(c) $-\frac{1}{2}$

(d) $\frac{1}{2}$

(e) $-\frac{5}{2}$

3. If $f'(x) = \frac{4}{\sqrt{1-x^2}}$ and $f\left(\frac{1}{2}\right) = 1$, then $1 - f\left(-\frac{1}{2}\right) =$

(a) $\frac{3\pi}{4}$

(b) $\frac{4\pi}{3}$

(c) $-\frac{3\pi}{4}$

(d) 0

(e) 2

4. Given $f(x) = 4x - 5$ and $\epsilon > 0$. The largest possible value of δ , such that $|f(x) - 7| < \epsilon$ whenever $|x - 3| < \delta$, is given by

(a) $\frac{\epsilon}{2}$

(b) $\frac{\epsilon}{4}$

(c) $\frac{\epsilon}{5}$

(d) ϵ

(e) $\frac{\epsilon}{3}$

5. Which one of the following statements is **TRUE** about the graph of the function

$$y = \frac{x}{x^2 - x - 2}?$$

- (a) It has two vertical asymptotes and no horizontal asymptote
- (b) It passes through the origin and has asymptotes at $x = 2$ and $x = -1$
- (c) It has two x -intercepts and one asymptote at $x = 0$
- (d) It has a y -intercept and one vertical asymptote
- (e) It passes through origin and has asymptotes at $x = 1$ and $x = \frac{-1}{2}$
6. If $y = \ln\left(\frac{x}{4}\right)$ and the value of x decreases from 4 to 3.9, then the corresponding change in y is approximated by the differential dy , which equals

(a) $-\ln\left(\frac{3.9}{4}\right)$

(b) $\frac{1}{100}$

(c) $\frac{1}{40}$

(d) $-\frac{1}{40}$

(e) $-\frac{1}{100}$

7. The graph of function $f(x) = x + 2 \sin x$ for $0 \leq x < 2\pi$, is concave up on the interval I and has inflection point $P(x, y)$ where

(a) $I = (\pi, 2\pi)$; $P(\pi, \pi)$

(b) $I = (\pi, 2\pi)$; $P(0, 0)$

(c) $I = (\pi, 2\pi)$; $P\left(\frac{\pi}{2}, \frac{\pi}{2} + 2\right)$

(d) $I = (0, \pi)$; $P(0, 0)$

(e) $I = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; $P(\pi, \pi)$

8. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, then the rate at which **the diameter** decreases, when the diameter is 10 cm, is

$$[\text{Hint: } (S = 4\pi r^2) \text{ and } (V = \frac{4}{3}\pi r^3)]$$

(a) $10\pi \text{ cm}/\text{min}$

(b) $20\pi \text{ cm}/\text{min}$

(c) $\frac{1}{10\pi} \text{ cm}/\text{min}$

(d) $\frac{1}{20\pi} \text{ cm}/\text{min}$

(e) $\frac{4}{30\pi} \text{ cm}/\text{min}$

9. If $y = (\ln x)^{\ln x}$, then

(a) $\frac{dy}{dx} = \left(\frac{\ln(\ln x) + 1}{x} \right) (\ln x)^{\ln x}$

(b) $\frac{dy}{dx} = \frac{1}{x} \ln(\ln x)$

(c) $\frac{dy}{dx} = \frac{\ln(\ln x + 1)}{x}$

(d) $\frac{dy}{dx} = \frac{1}{x} (\ln x)^{\ln x - 1}$

(e) $\frac{dy}{dx} = (\ln x)^{\ln x - 1}$

10. Let g be a twice differentiable function. If $f(x) = g(\sqrt{x})$, then $f''(x)$ is equal to

(a) $\frac{1}{4x} g''(\sqrt{x}) - \frac{1}{4\sqrt{x^3}} g'(\sqrt{x})$

(b) $\frac{1}{2\sqrt{x}} g''(\sqrt{x}) - \frac{1}{4x}$

(c) $\frac{1}{4x} g''(\sqrt{x})$

(d) $\frac{-1}{4\sqrt{x^3}} g''(\sqrt{x})$

(e) $\frac{1}{2\sqrt{x}} g''(\sqrt{x})$

11. The number of the **different equations** of the normal lines of slope 1 to the graph of $y = \frac{1}{x+1}$ is

(a) 0

(b) 1

(c) 3

(d) 4

(e) 2

12. The function $f(x) = x^4(x-1)^3$ has

(a) three local maxima and four local minima

(b) one local maximum and two local minima

(c) two local maxima and two local minima

(d) two local maxima and one local minimum

(e) one local maximum and one local minimum

13. The value(s) of constant c making the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

- (a) -2 and 2
 - (b) 4 and -4
 - (c) 4
 - (d) 20
 - (e) -2
14. $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$ is equal to

- (a) 3
- (b) $-\frac{3}{2}$
- (c) $\frac{3}{2}$
- (d) 1
- (e) -1

15. The slope of the tangent line to the graph of $x^2y^2 + x \sin y = 4$ at the point $\left(\frac{2}{\pi}, \pi\right)$ is equal to

(a) $-\frac{2}{3}\pi^2$

(b) $\frac{3}{2}\pi$

(c) $-\frac{1}{2}\pi^2$

(d) $\frac{1}{2}\pi^2$

(e) $-\frac{3}{2}\pi$

16. Given $f(x) = \frac{x-1}{x^2}$, $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{2(x-3)}{x^4}$, Then which one of the following statements is **FALSE** about the graph of the function $f(x)$.

[Hint: Sketch]

(a) the graph intersects its horizontal asymptote

(b) the graph is concave up on $(2, 3)$

(c) the graph has one local maximum

(d) the graph is concave up on $(3, \infty)$

(e) the graph has one inflection point

17. The x-coordinate of the point at which the tangent line to the curve $y = \cosh(2x)$ has slope 2, is equal to

(a) $3\ln(1 + \sqrt{2})$

(b) $2\ln(1 + \sqrt{2})$

(c) $\frac{2}{3}\ln(1 + \sqrt{2})$

(d) $\frac{3}{2}\ln(1 + \sqrt{2})$

(e) $\frac{1}{2}\ln(1 + \sqrt{2})$

18. The **height** of the right circular cylinder of largest volume which is inscribed in a sphere of radius 9 cm is

(a) $9\sqrt{3}$ cm

(b) $6\sqrt{3}$ cm

(c) $\frac{9\sqrt{3}}{2}$ cm

(d) $\frac{5\sqrt{3}}{2}$ cm

(e) $4\sqrt{3}$ cm

19. $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1+\ln x)}$ is equal to

- (a) 2
- (b) 1
- (c) $1 + \ln 2$
- (d) 0
- (e) $\ln 2$

20. The values for f, g, f', g' are given by the table below:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	9	8	5
4	-3	3	2	-6

Then $\frac{d}{dx}(g(x + f(x)))$ at $x = 3$ is

- (a) -54
- (b) -6
- (c) 27
- (d) -48
- (e) 5

21. If M and m are, respectively, the absolute maximum and minimum of $f(x) = xe^{-x}$ on the interval $[0, 2]$, then $e^2M - em =$
- (a) 2
 - (b) e
 - (c) $-e$
 - (d) $2e$
 - (e) $\frac{e+1}{e}$
22. Which of the following statements is **TRUE** about $f(x) = x^{2/3}$?
- (a) f has no vertical tangent line
 - (b) f has a vertical tangent line at $(0, 0)$
 - (c) f has a vertical tangent line at $x = 1$
 - (d) f is differentiable on $(-\infty, \infty)$
 - (e) f has a horizontal tangent line

23. Starting with $x_1 = 1$, the next approximation x_2 to a root of $\tan^{-1} x = 1 - x$ by Newton's method is

[Use $\pi = \frac{22}{7}$]

- (a) $\frac{8}{21}$
- (b) $\frac{14}{21}$
- (c) $\frac{13}{21}$
- (d) $\frac{10}{21}$
- (e) $\frac{19}{21}$

24. The picture shows the graphs of f , f' and f'' . Identify each curve.

- (a) $f = A$, $f' = B$, $f'' = C$
- (b) $f = C$, $f' = B$, $f'' = A$
- (c) $f = B$, $f' = C$, $f'' = A$
- (d) $f = B$, $f' = A$, $f'' = C$
- (e) $f = C$, $f' = A$, $f'' = B$

25. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then which of the following statements is **TRUE**?

[Hint: Apply the Mean Value Theorem]

- (a) $f(4) \geq 16$
- (b) $f(4) \leq 10$
- (c) $f(4) = 15$
- (d) $f(4) = 0$
- (e) $f(4) < 16$

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

CODE 004

**Math 101
Final Exam
061**

CODE 004

**Wednesday 24/1/2007
Net Time Allowed: 150 minutes**

Name: _____

ID: _____ Sec: _____

Check that this exam has 25 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The function $f(x) = x^4(x - 1)^3$ has
- (a) one local maximum and two local minima
 - (b) three local maxima and four local minima
 - (c) one local maximum and one local minimum
 - (d) two local maxima and one local minimum
 - (e) two local maxima and two local minima
2. Which one of the following statements is **TRUE** about the graph of the function
- $$y = \frac{x}{x^2 - x - 2}?$$
- (a) It passes through origin and has asymptotes at $x = 1$ and $x = \frac{-1}{2}$
 - (b) It has a y -intercept and one vertical asymptote
 - (c) It has two x -intercepts and one asymptote at $x = 0$
 - (d) It passes through the origin and has asymptotes at $x = 2$ and $x = -1$
 - (e) It has two vertical asymptotes and no horizontal asymptote

3. $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$ is equal to

(a) 1

(b) $\frac{3}{2}$

(c) -1

(d) 3

(e) $-\frac{3}{2}$

4. Let g be a twice differentiable function. If $f(x) = g(\sqrt{x})$, then $f''(x)$ is equal to

(a) $\frac{1}{4x}g''(\sqrt{x})$

(b) $\frac{-1}{4\sqrt{x^3}}g''(\sqrt{x})$

(c) $\frac{1}{2\sqrt{x}}g''(\sqrt{x})$

(d) $\frac{1}{2\sqrt{x}}g''(\sqrt{x}) - \frac{1}{4x}$

(e) $\frac{1}{4x}g''(\sqrt{x}) - \frac{1}{4\sqrt{x^3}}g'(\sqrt{x})$

5. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$, then $\left(\frac{f}{f-g}\right)'(3)$ is equal to

(a) 12

(b) 32

(c) 8

(d) 44

(e) 16

6. If $f'(x) = \frac{4}{\sqrt{1-x^2}}$ and $f\left(\frac{1}{2}\right) = 1$, then $1 - f\left(-\frac{1}{2}\right) =$

(a) 2

(b) 0

(c) $-\frac{3\pi}{4}$

(d) $\frac{3\pi}{4}$

(e) $\frac{4\pi}{3}$

7. The slope of the tangent line to the graph of $x^2y^2 + x \sin y = 4$ at the point $\left(\frac{2}{\pi}, \pi\right)$ is equal to

(a) $\frac{1}{2}\pi^2$

(b) $-\frac{3}{2}\pi$

(c) $-\frac{1}{2}\pi^2$

(d) $\frac{3}{2}\pi$

(e) $-\frac{2}{3}\pi^2$

8. If the line $2x + y = b$ is tangent to the parabola $y = ax^2$ at $x = 2$, then $a - b =$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{3}{2}$

(d) $-\frac{3}{2}$

(e) $-\frac{5}{2}$

9. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$, then the rate at which **the diameter** decreases, when the diameter is 10 cm, is

$$[\text{Hint: } (S = 4\pi r^2) \text{ and } (V = \frac{4}{3}\pi r^3)]$$

- (a) $20\pi \text{ cm}/\text{min}$
- (b) $\frac{1}{20\pi} \text{ cm}/\text{min}$
- (c) $\frac{4}{30\pi} \text{ cm}/\text{min}$
- (d) $\frac{1}{10\pi} \text{ cm}/\text{min}$
- (e) $10\pi \text{ cm}/\text{min}$
10. If $y = \ln\left(\frac{x}{4}\right)$ and the value of x decreases from 4 to 3.9, then the corresponding change in y is approximated by the differential dy , which equals

(a) $-\ln\left(\frac{3.9}{4}\right)$

(b) $-\frac{1}{100}$

(c) $\frac{1}{100}$

(d) $\frac{1}{40}$

(e) $-\frac{1}{40}$

11. The value(s) of constant c making the function

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

continuous on $(-\infty, \infty)$ is (are)

- (a) -2 and 2
 - (b) -2
 - (c) 20
 - (d) 4
 - (e) 4 and -4
12. The graph of function $f(x) = x + 2 \sin x$ for $0 \leq x < 2\pi$, is concave up on the interval I and has inflection point $P(x, y)$ where

- (a) $I = (\pi, 2\pi)$; $P\left(\frac{\pi}{2}, \frac{\pi}{2} + 2\right)$
- (b) $I = (0, \pi)$; $P(0, 0)$
- (c) $I = (\pi, 2\pi)$; $P(0, 0)$
- (d) $I = (\pi, 2\pi)$; $P(\pi, \pi)$
- (e) $I = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; $P(\pi, \pi)$

13. The number of the **different equations** of the normal lines of slope 1 to the graph of $y = \frac{1}{x+1}$ is

(a) 3

(b) 1

(c) 2

(d) 4

(e) 0

14. If $y = (\ln x)^{\ln x}$, then

(a) $\frac{dy}{dx} = \left(\frac{\ln(\ln x) + 1}{x} \right) (\ln x)^{\ln x}$

(b) $\frac{dy}{dx} = \frac{1}{x} (\ln x)^{\ln x - 1}$

(c) $\frac{dy}{dx} = (\ln x)^{\ln x - 1}$

(d) $\frac{dy}{dx} = \frac{\ln(\ln x + 1)}{x}$

(e) $\frac{dy}{dx} = \frac{1}{x} \ln(\ln x)$

15. Given $f(x) = 4x - 5$ and $\epsilon > 0$. The largest possible value of δ , such that $|f(x) - 7| < \epsilon$ whenever $|x - 3| < \delta$, is given by

(a) $\frac{\epsilon}{3}$

(b) $\frac{\epsilon}{4}$

(c) $\frac{\epsilon}{5}$

(d) $\frac{\epsilon}{2}$

(e) ϵ

16. The **height** of the right circular cylinder of largest volume which is inscribed in a sphere of radius 9 cm is

(a) $6\sqrt{3}$ cm

(b) $9\sqrt{3}$ cm

(c) $\frac{9\sqrt{3}}{2}$ cm

(d) $4\sqrt{3}$ cm

(e) $\frac{5\sqrt{3}}{2}$ cm

17. The x-coordinate of the point at which the tangent line to the curve $y = \cosh(2x)$ has slope 2, is equal to

(a) $2\ln(1 + \sqrt{2})$

(b) $3\ln(1 + \sqrt{2})$

(c) $\frac{1}{2}\ln(1 + \sqrt{2})$

(d) $\frac{3}{2}\ln(1 + \sqrt{2})$

(e) $\frac{2}{3}\ln(1 + \sqrt{2})$

18. $\lim_{x \rightarrow \infty} x^{(\ln 2)/(1 + \ln x)}$ is equal to

(a) 2

(b) 0

(c) $1 + \ln 2$

(d) $\ln 2$

(e) 1

19. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then which of the following statements is **TRUE**?

[Hint: Apply the Mean Value Theorem]

- (a) $f(4) \leq 10$
- (b) $f(4) = 15$
- (c) $f(4) \geq 16$
- (d) $f(4) = 0$
- (e) $f(4) < 16$

20. Starting with $x_1 = 1$, the next approximation x_2 to a root of $\tan^{-1} x = 1 - x$ by Newton's method is

[Use $\pi = \frac{22}{7}$]

- (a) $\frac{13}{21}$
- (b) $\frac{10}{21}$
- (c) $\frac{8}{21}$
- (d) $\frac{19}{21}$
- (e) $\frac{14}{21}$

21. Which of the following statements is **TRUE** about $f(x) = x^{2/3}$?
- (a) f has a vertical tangent line at $x = 1$
 - (b) f is differentiable on $(-\infty, \infty)$
 - (c) f has no vertical tangent line
 - (d) f has a horizontal tangent line
 - (e) f has a vertical tangent line at $(0, 0)$

22. The values for f, g, f', g' are given by the table below:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	1	9	8	5
4	-3	3	2	-6

Then $\frac{d}{dx}(g(x + f(x)))$ at $x = 3$ is

- (a) -48
- (b) 5
- (c) -54
- (d) -6
- (e) 27

23. Given $f(x) = \frac{x-1}{x^2}$, $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{2(x-3)}{x^4}$, Then which one of the following statements is **FALSE** about the graph of the function $f(x)$.

[Hint: Sketch]

- (a) the graph is concave up on $(2, 3)$
 - (b) the graph has one local maximum
 - (c) the graph has one inflection point
 - (d) the graph is concave up on $(3, \infty)$
 - (e) the graph intersects its horizontal asymptote
24. If M and m are, respectively, the absolute maximum and minimum of $f(x) = xe^{-x}$ on the interval $[0, 2]$, then $e^2M - em =$
- (a) $-e$
 - (b) 2
 - (c) e
 - (d) $\frac{e+1}{e}$
 - (e) $2e$

25. The picture shows the graphs of f , f' and f'' . Identify each curve.

(a) $f = B$, $f' = C$, $f'' = A$

(b) $f = B$, $f' = A$, $f'' = C$

(c) $f = C$, $f' = A$, $f'' = B$

(d) $f = A$, $f' = B$, $f'' = C$

(e) $f = C$, $f' = B$, $f'' = A$

Q	MM	V1	V2	V3	V4
1	a	b	b	c	c
2	a	c	c	e	d
3	a	a	d	b	e
4	a	a	c	b	e
5	a	a	c	b	c
6	a	b	b	d	e
7	a	b	b	a	e
8	a	b	e	d	e
9	a	b	d	a	b
10	a	a	e	a	e
11	a	c	b	b	b
12	a	e	c	e	d
13	a	d	a	e	b
14	a	d	d	b	a
15	a	e	c	a	b
16	a	a	e	b	a
17	a	d	c	e	c
18	a	d	c	b	a
19	a	c	b	a	c
20	a	a	a	a	b
21	a	d	a	b	e
22	a	e	e	b	c
23	a	d	d	d	a
24	a	c	c	b	c
25	a	e	b	a	e