

## 4.8 Newton method.

a method to find zeros of a function. i.e. if we  
use some initial point  $x_1$ , then equation of the tangent

$$y - f(x_1) \approx f'(x_1)(x - x_1)$$

If  $f'(x_1) \neq 0$  then the tangent crosses the  $x$ -axis at  $(x_2, 0)$

$$0 - f(x_1) \approx f'(x_1)(x_2 - x_1)$$

$$\text{Solve for } x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Newton method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Ex1

Approx. to 4-d the root of  $y = x^5 + x^2 - 9x - 3$  in  $[-2, -1]$

$$x_{n+1} = x_n - \frac{x_n^5 + x_n^2 - 9x_n - 3}{5x_n^4 + 2x_n - 9}$$

$$x_1 = -1 \quad x_2 = -1 - \frac{[-1]^5 + (-1)^2 - 9(-1) - 3}{[5(-1)^4 + 2(-1) - 9]} = 0$$

$$x_2 = 0 \quad x_3 = 0 - \frac{-3}{-9} = 0.3333 \quad x_4 = -0.322193$$

$$x_5 = -0.3222 \quad \text{out side range}$$

$$x_1 = -2 \quad x_2 = -1.8060 \quad x_3 = -1.7395$$

$$x_4 = -1.7321 \quad x_5 = -1.7321$$

Ex2. Approx. up to 4-d  $\sqrt{57}$  An.  $f(x) = x^2 - 57$   $f' = 2x$

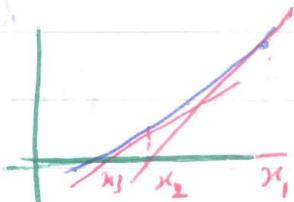
$$x_{n+1} = x_n - \frac{x_n^2 - 57}{2x_n}$$

$$x_1 = 8 \quad x_2 = 8 - \frac{8^2 - 57}{16} = 7.5625 \quad x_3 = 7.549845$$

$$x_4 = 7.549844$$

d-4.

calculator 7.54983481



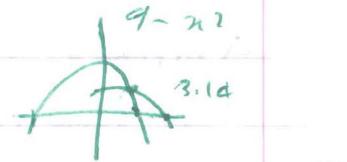
(101)

Ex 3 Approximate the point of intersection of

$$y = \cos \frac{x}{2} \quad y = 9 - x^2$$

$$f(x) = \cos \frac{x}{2} + x^2 - 9$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad x_1 = 3 \quad x_2 = 2.9821116 \quad x_3 = 2.9821117$$

Ex 4

Difficulties

$$f(x) = \sqrt[3]{x}$$

$$x_{n+1} = x_n - \frac{x_n}{\frac{1}{3} x_n^{-2/3}}$$

$$= x_n - 3x_n = -2x_n$$



$$x_1 = 1 \quad x_2 = -2 \quad x_3 = 4 \quad x_5 = -8$$

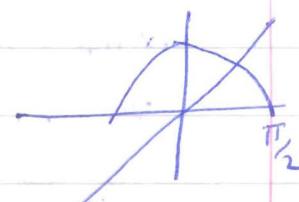
$$\text{Ex. } x - \cos x = 0$$

Find  $x$ 

$$f \quad x_{n+1} = x_n - \frac{x_n - \cos x_n}{1 + \sin x_n}$$

$$x_1 = \cancel{0.5} \frac{\pi}{4}$$

$$x_2 = \cancel{0.5} - \frac{\cancel{0.5} - \cos \frac{\pi}{4}}{1 + \sin \frac{\pi}{4}}$$



$$x_3 = \dots$$

## Antiderivatives

Def A function  $F$  is an antiderivative of  $f$  if  $F'(x) = f(x)$ .

Ex. Find  $F(x)$  (antiderivative)

$$(a) f(x) = 2x$$

$$F(x) = x^2 + C$$

$$(b) g(x) = \cos x$$

$$F(x) = \sin x + C$$

$$(c) h(x) = \frac{1}{x} + 2e^{2x}$$

$$H(x) = \ln|x| + e^{2x} + C.$$

If  $F$  is antiderivative of  $f$   
then the general antiderivative of  $f$  is  
 $F(x) + C$ .

trigonometric functions +

The general antiderivative is  
called indefinite integral

$$F(x) = \int f(x) dx$$

$$\text{Ex. } \int 2x dx = x^2 + C = F(x)$$