

Math101, Quiz # 3

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Sec: 25

(a) $\frac{dy}{dx}$

1. Find $\lim_{t \rightarrow \infty} t - t^2$, then $\lim_{t \rightarrow \infty} 2 \tan^{-1}(t - t^2)$.

$$\lim_{t \rightarrow \infty} t - t^2 \Rightarrow \infty - (\infty)^2 = \underline{-\infty} \quad 3 //$$

$$\lim_{t \rightarrow \infty} 2 \tan^{-1}(t - t^2) \Rightarrow \lim_{t \rightarrow \infty} 2 \tan^{-1}(\infty) =$$

2. Use the Intermediate Value Theorem to show that the equation $e^{-x} = 2x$ has a root in the interval $(0,1)$.

$$f(x) = 2x - e^{-x}$$

$$\textcircled{1} \quad P(0) = 2 \cdot 0 - e^0 = -1 < 0$$

$$\textcircled{2} \quad P(1) = 2 \cdot 1 - e^{-1} = 2 - \frac{1}{e} = +\text{num} > 0 \quad \checkmark$$

from \textcircled{1}, \textcircled{2} we find that there is a point $P(c)$

where the value of ~~f(x)~~ $f(x) = 0$

3. Find the vertical and horizontal asymptotes of the curve $y = \frac{x^2+2}{x^3+x}$.

using limits

$$\lim_{x \rightarrow \infty} \frac{x^2+2}{x^3+x} = \lim_{x \rightarrow \infty} \frac{x^{2/3}/x + 2/x^3}{x^{3/3}/x + 1/x^2}$$

$$\lim_{x \rightarrow \infty} \frac{x^{2/3}/x + 2/x^3}{x^{3/3}/x + 1/x^2} \quad \checkmark$$

$$\lim_{x \rightarrow \infty} \frac{\cancel{x^2} + \cancel{2}}{\cancel{x^3} + \cancel{x}} = \frac{0+0}{1+0} = \underline{0}$$

4. Find the equation of the tangent line to the curve $y = \sqrt[3]{1 + \tan(3x)}$ at the point $(0,1)$.

$$x=0 \quad y=1$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \lim_{h \rightarrow 0} \frac{(1 + \tan 3(x+h))^{1/3} - (1 + \tan 3x)^{1/3}}{h} \quad x=0/$$

$$\frac{(1 + \tan 3h)^{1/3} - (1 + \tan 3h)^{1/3}}{h} \Rightarrow \frac{0-0}{0} = \boxed{1} \quad \text{equation}$$

$$\frac{y-1}{x-0} = 1$$

$$y-1 = x$$

$$\text{eq. } \underline{y = x+1}$$