

⑤ $\min -2x_1 + x_2 - x_3$
 s.t $x_1 + x_2 + x_3 \leq 6 \rightarrow x_1 + x_2 + x_3 + x_4 = 6$
 $-x_1 + 2x_2 \leq 4 \rightarrow -x_1 + 2x_2 + x_5 = 4$
 $x_i \geq 0$

x_4	①	1	1	1	0	6
x_5		-1	2	0	0	4
		-2	1	-1	0	0

x_4 will leave the base
 x_1 will enter the base

x_1	1	1	1	1	0	6
x_5	0	3	1	1	1	10
	0	3	1	2	0	12

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$
 $Z = -12$

⑦ $\min -3x_1 - 2x_2$
 s.t $3x_1 + 4x_2 \leq 5 \rightarrow 3x_1 + 4x_2 + x_3 = 5$
 $2x_1 + x_2 \geq 2 \rightarrow 2x_1 + x_2 - x_4 = 2$
 $-x_1 + x_2 \geq 1 \rightarrow -x_1 + x_2 - x_5 = 1$
 $x_i \geq 0$

Phase 01
 with 5 rows
 $3x_1 + 4x_2 + x_3 = 5$
 $2x_1 + x_2 - x_4 = 2$
 $-x_1 + x_2 - x_5 = 1$

Phase 01

x_3	3	4	1	0	0	0	5
y_1	2	1	0	-1	0	1	2
y_2	-1	1	0	0	-1	0	1
	0	0	0	0	0	1	0

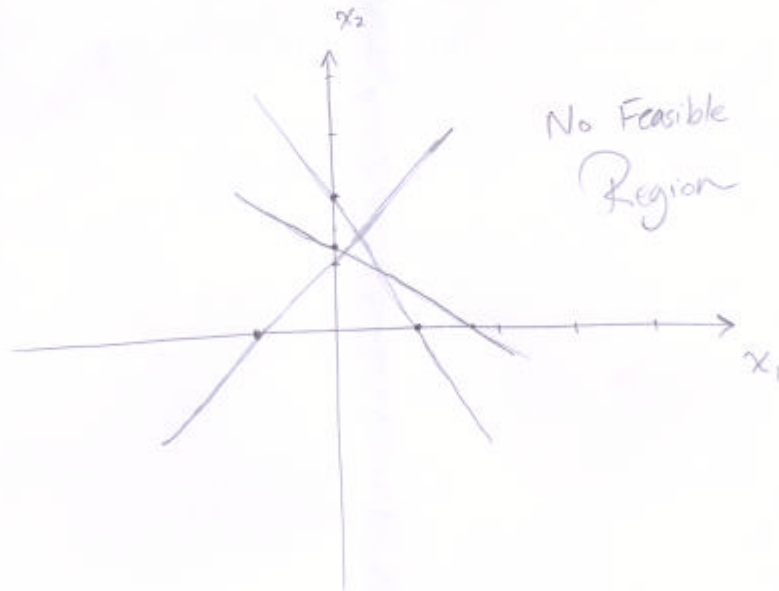
x_3	3	4	1	0	0	0	5
y_1	2	1	0	-1	0	1	2
y_2	-1	1	0	0	-1	0	1
	-1	②	0	1	1	0	-3

x_2 will enter
 y_2 will leave

x_3	7	0	1	0	4	0	-4	1	x_1 will enter
y_1	3	0	0	-1	1	1	-1	1	x_3 will leave
x_2	-1	1	0	0	-1	0	1	1	
⊖	0	0	1	-1	0	0	2	-1	

x_1	1	0	$1/2$	0	$4/2$	0	$-4/2$	$1/2$
y_1	0	0	$-3/2$	-1	$-5/2$	1	$5/2$	$4/2$
x_2	0	1	$1/2$	0	$-3/2$	0	$3/2$	$2/2$
	0	0	$3/2$	1	$5/2$	0	1	$-4/2$

Since $r_j \geq 0$ but $Z \neq 0 \Rightarrow$ the feasible region is empty.



(14) $\min Z = -2x_1 + 3x_2 - 4x_3$

s.t. $x_1 - 2x_2 + 4x_3 \leq 15 \rightarrow x_1 - 2x_2 + 4x_3 + x_4 = 15$

$2x_1 - x_2 + 5x_3 \leq 10 \rightarrow 2x_1 - x_2 + 5x_3 + x_5 = 10$

$x_i \geq 0$

x_4 1 -2 4 1 0 15

x_5 2 -1 5 0 1 10

(-2) 3 -4 0 0 0

Since $r_1 = -2$ is the first negative, x_1 will enter the base.

x_5 will leave the base.

x_4 0 $-\frac{3}{2}$ $\frac{3}{2}$ 1 $-\frac{1}{2}$ 10

x_1 1 $-\frac{1}{2}$ $\frac{5}{2}$ 0 $\frac{1}{2}$ 5

0 2 1 0 1 10

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$Z = -10$$

$$\textcircled{22} \quad \max \quad 5x_1 + 6x_2 + 9x_3 + 8x_4$$

$$\text{s.t.} \quad x_1 + 2x_2 + 3x_3 + x_4 \leq 5$$

$$x_1 + x_2 + 2x_3 + 3x_4 \leq 3$$

$$x_i \geq 0$$

$$\min \quad -5x_1 - 6x_2 - 9x_3 - 8x_4$$

$$\text{s.t.} \quad x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 5$$

$$x_1 + x_2 + 2x_3 + 3x_4 + x_6 = 3$$

$$d^T = C_B^T B^{-1} = [0 \ 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0 \ 0]$$

$$r_N^T = C_N^T - d^T N = C_N^T = [-5 \ -6 \ -9 \ -8]$$

x_3 will enter the base.

$$a_3^* = B^{-1} a_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$r_3 = c_3 - d^T a_3 = -9 - 0 = -9$$

	B^{-1}	b	a_3	
x_5	1	0	5	3
x_6	0	1	3	$\textcircled{2}$
	0	0	0	-9

x_6 will leave the base

x_5	1	$-\frac{3}{2}$	$\frac{1}{2}$	0
x_3	0	$\frac{1}{2}$	$\frac{3}{2}$	1
	0	$\frac{9}{2}$	$\frac{27}{2}$	0

$$d^T = c_B^T B^{-1} = [0 \quad -9] \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = [0 \quad -\frac{9}{2}]$$

$$r_1 = c_1 - d^T a_1 = -5 - [0 \quad -\frac{9}{2}] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + \frac{9}{2} = -\frac{1}{2}$$

$$r_2 = c_2 - d^T a_2 = -6 - [0 \quad -\frac{9}{2}] \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -6 + \frac{9}{2} = -\frac{3}{2}$$

$$r_4 = c_4 - d^T a_4 = -8 - [0 \quad -\frac{9}{2}] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -8 + \frac{18}{2} = \frac{11}{2}$$

$$r_6 = c_6 - d^T a_6 = 0 - [0 \quad -\frac{9}{2}] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + \frac{9}{2} = \frac{9}{2}$$

x_2 will enter the base

$$a_2^* = B^{-1} a_2 = \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{l} x_5 \\ x_3 \end{array} \begin{array}{cccc} 1 & -\frac{3}{2} & \frac{1}{2} & (\frac{1}{2}) \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{9}{2} & \frac{23}{2} & \frac{13}{2} \end{array}$$

x_5 will leave the base

$$\begin{array}{l} x_2 \\ x_3 \\ \end{array} \begin{array}{cccc} 2 & -3 & 1 & 1 \\ -1 & 2 & 1 & 0 \\ 3 & 0 & 15 & 0 \end{array}$$

$$d^T = c_B^T B^{-1} = [-6 \quad -9] \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = [-3 \quad 0]$$

$$r_1 = c_1 - d^T a_1 = -5 - [-3 \quad 0] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + 3 = -2$$

$$r_4 = c_4 - d^T a_4 = -8 - [-3 \quad 0] \begin{bmatrix} 1 \\ 2 \end{bmatrix} = -8 + 3 = -5$$

$$r_5 = c_5 - d^T a_5 = 0 - [-3 \quad 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 + 3 = 3$$

$$r_6 = c_6 - d^T a_6 = 0 - [-3 \quad 0] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + 0 = 0$$

x_6 will enter the base

$$a_4^* = B^{-1} a_4 = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

$$x_2 \quad 2 \quad -3 \quad 1 \quad -7$$

$$x_3 \quad -1 \quad 2 \quad 1 \quad \textcircled{5}$$

$$3 \quad 0 \quad 19 \quad -5$$

x_3 will leave the base.

$$x_2 \quad \frac{3}{5} \quad -\frac{1}{5} \quad \frac{13}{5} \quad 0$$

$$x_4 \quad -\frac{1}{5} \quad \frac{2}{5} \quad \frac{1}{5} \quad 1$$

$$2 \quad 2 \quad 16 \quad 0$$

$$d^T = C_B^T B^{-1} = [-6 \quad -8] \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} = \begin{bmatrix} -2 & -2 \end{bmatrix}$$

$$r_1 = c_1 - d^T a_1 = -5 - [-2 \quad -2] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -5 + 4 = -1$$

$$r_3 = c_3 - d^T a_3 = -9 - [-2 \quad -2] \begin{bmatrix} 3 \\ 2 \end{bmatrix} = -9 + 10 = 1$$

$$r_5 = c_5 - d^T a_5 = 0 - [-2 \quad -2] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2$$

$$r_6 = c_6 - d^T a_6 = 0 - [-2 \quad -2] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2$$

x_1 will enter the base

$$a_1^* = B^{-1} a_1 = \begin{bmatrix} \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \end{bmatrix}$$

$$\begin{array}{ccccc}
 x_2 & x_3 & -x_5 & 12x_6 & 2/5 \\
 x_4 & -1/5 & 3/5 & 1/5 & \textcircled{2/5} \\
 & 2 & 2 & 16 & -1
 \end{array}$$

x_4 will leave the base.

$$\begin{array}{ccccc}
 & & \theta^+ & & b \\
 x_2 & 1 & -1 & 2 & 0 \\
 x_1 & -1 & 2 & 1 & 1 \\
 & 1 & 4 & 17 & 0
 \end{array}$$

$$d^T = C_B^T B^{-1} = [-6 \quad -5] \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -4 \end{bmatrix}$$

$$r_3 = c_3 - d^T a_3 = -4 - [-1 \quad -4] \begin{bmatrix} 3 \\ 2 \end{bmatrix} = -4 + 11 = 7$$

$$r_4 = c_4 - d^T a_4 = -8 - [-1 \quad -4] \begin{bmatrix} 1 \\ 3 \end{bmatrix} = -8 + 13 = 5$$

$$r_5 = c_5 - d^T a_5 = 0 - [-1 \quad -4] \begin{bmatrix} 1 \\ 6 \end{bmatrix} = 0 + 1 = 1$$

$$r_6 = c_6 - d^T a_6 = 0 - [-1 \quad -4] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 + 4 = 4$$

Since all $r_n \geq 0 \Rightarrow$ OBFS $\rightarrow \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $z = -17$

