Solution HW #1

HW #1

Let x_1 be the number of tons of shoestring potatoes and x_2 the number of tons of nuggets processed each day.

Maximize:	Subject to:	
$z = 60x_1 + 45x_2$	$x_1 + x_2 \le 8$	(Peeling)
	$\frac{5}{2}x_1 + \frac{3}{2}x_2 \le 16$	(Slicing)
	$x_1, x_2 \ge 0$	

Let x_1, x_2, x_3 be the amounts invested in low-risk, medium-risk, and high-risk stocks, respectively.

Maximize:	Subject to:
$z = 0.07x_1 + 0.09x_2 + 0.11x_3$	$x_1 - x_3 \le 2000$
	$x_3 \le 8000$
	$x_2 + x_3 \le 14{,}000$
	$x_1 + x_2 + x_3 = 18{,}000$
	$x_i \ge 0, \quad 1 \le i \le 3$

5. Let L be acres in lawn, and let T, B, P, C_1, and C_2 be acres of tomatoes, beans, peas, corn, and carrots, respectively.

Maximize:	
$z = 750T - 150T$	(Tomatoes)
$+ 150B - 40B$	(Beans)
$+ 160P - 30P$	(Peas)
$+ 50C_1 - 25C_1$	(Corn)
$+ 180C_2 - 55C_2$	(Carrots)

Subject to:

$90T + 18B + 27P + 10C_1 + 30C_2 + 6L \le 300$	(Labor)
$150T + 40B + 30P + 25C_1 + 55C_2 + 15L \le 400$	(Capital)
$T + B + P + C_1 + C_2 + L = 4$	(Land)
$T, B, P, C_1, C_2, L \ge 0$	

7. Let a, b, c, d, e be the respective amounts of products A, B, C, D, E that are produced. Measure time in minutes.

(a) Maximize:

$z = 12a + 11b + 12c + \frac{21}{2}d + 6e$	(Selling price)
$-2a - 2b - 2c - d - e$	(Materials cost)
$-\frac{135}{60}a - \frac{72}{60}b - \frac{72}{60}c - \frac{108}{60}d - \frac{81}{60}e$	(M_1 cost)
$-\frac{72}{60}a - \frac{90}{60}b - \frac{108}{60}c - \frac{36}{60}d - \frac{54}{60}e$	(M_2 cost)
$-\frac{72}{60}a - \frac{108}{60}b - \frac{120}{60}c - \frac{144}{60}d$	(M_3 cost)

(b) $c \ge 20$ (Minimum order)

$d \ge 30$	
$15a + 8b + 8c + 12d + 9e \le 4800$	(M_1)
$8a + 10b + 12c + 4d + 6e \le 4800$	(M_2)
$6a + 9b + 10c + 12d \le 4800$	(M_3)
$a, b, c, d, e \ge 0$	

9. Let a, b, c, d, e be the respective amounts of products A, B, C, D, E. Measure time in minutes. Let c_1 be the number of units of C in excess of 20, and let d_1 be the number of units of D in excess of 30.

Maximize:

$z = 12a + 11b + 12c + \frac{21}{2}d + 6e + 9c_1 + 8d_1$	
$-2a - 2b - 2c - d - e - 2c_1 - d_1$	
$-\frac{135}{60}a - \frac{72}{60}b - \frac{72}{60}c - \frac{108}{60}d - \frac{81}{60}e - \frac{72}{60}c_1 - \frac{108}{60}d_1$	(Extra sales)
$-\frac{72}{60}a - \frac{90}{60}b - \frac{108}{60}c - \frac{36}{60}d - \frac{54}{60}e - \frac{108}{60}c_1 - \frac{36}{60}d_1$	(Extra sales)
$-\frac{72}{60}a - \frac{108}{60}b - \frac{120}{60}c - \frac{144}{60}d - \frac{120}{60}c_1 - \frac{144}{60}d_1$	(Extra sales)

Subject to:

$c = 20$ (or c could be replaced throughout by 20)
$d = 30$ (or d could be replaced throughout by 30)
$15a + 8b + 8c + 12d + 9e + 8c_1 + 12d_1 \le 4800$
$8a + 10b + 12c + 4d + 6e + 12c_1 + 4d_1 \le 4800$
$6a + 9b + 10c + 12d + 10c_1 + 12d_1 \le 4800$

Since the cost coefficients of x_1, x_2, x_3 are in descending order, we should try to make x_1, x_2, x_3 large in that order. Since $x_1 + x_2 \le x_3$, it follows that $x_1 + x_2 \le 50{,}000$ (if $x_1 + x_2 > 50{,}000$, then $x_1 + x_2 + x_3 > 100{,}000$). We obtain the largest contribution to the objective function by setting $x_1 = (50{,}000)/3$ and $x_2 = (\frac{2}{3})(50{,}000)$. It follows that $x_3 = 50{,}000$.

Let m_1, m_2, m_3 be the number of pounds of each mineral in a 100-lb batch.

Minimize: $$z = \frac{35}{10}m_1 + \frac{25}{10}m_2 + 3m_3$$

Subject to:

$$\begin{aligned}
\frac{3}{100}m_1 + \frac{7}{100}m_2 + \frac{9}{100}m_3 &\ge 4 && \text{(A)}\\
\frac{5}{100}m_1 + \frac{8}{100}m_2 + \frac{1}{100}m_3 &\ge 3 && \text{(B)}\\
\frac{35}{100}m_1 + \frac{32}{100}m_2 + \frac{27}{100}m_3 &\ge 30 && \text{(C)}\\
\frac{24}{100}m_1 + \frac{12}{100}m_2 + \frac{15}{100}m_3 &\ge 16 && \text{(D)}\\
-\frac{1}{100}m_1 + \frac{99}{100}m_2 - \frac{1}{100}m_3 &\le 0 &&\\
m_1 + \quad m_2 + \quad m_3 &= 100 &&\\
m_i \ge 0, \quad 1 \le i \le 3 & &&
\end{aligned}$$

Note that the fifth inequality is equivalent to the constraint

$$m_2 \le 0.01(m_1 + m_2 + m_3)$$