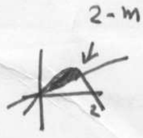
$\boxed{1}$ $A = \int_0^2 2x - x^2\,dx = \frac{4}{3}$

$$\frac{A}{2} = \int_0^{2-m} 2x - x^2 - mx\,dx = \frac{1}{6}(2-m)^3 = \frac{A}{2} = \frac{4/3}{2} = \frac{4}{6}$$



$$\Rightarrow (2-m)^3 = 4 \Rightarrow 2-m = \sqrt[3]{4} \Rightarrow m = 2 - \sqrt[3]{4}.$$

$\boxed{2}$ disks $V = \pi \int_{-1}^{1} [1-y^2]^2\,dy = \pi \int_{-1}^{1} 1 - 2y^2 + y^4\,dy$

$$= \pi \left[y - \frac{2y^3}{3} + \frac{y^5}{5}\right]_{-1}^{1} = \pi\,\frac{16}{15}$$

shells $V = 2\pi \int_0^1 x\left[\sqrt{1-x} - (-\sqrt{1-x})\right]dx$

$$= 2\pi \int_0^1 x\,2\sqrt{1-x}\,dx = 4\pi \int_0^1 x\sqrt{1-x}\,dx \qquad \begin{array}{l} u = 1-x \\ du = -dx \\ x = 1-u \end{array}$$

$$= +4\pi \int_1^0 (1-u)\sqrt{u}\,(-du) = 4\pi \int_0^1 (1-u)\sqrt{u}\,du$$

$$= 4\pi \int_0^1 u^{1/2} - u^{3/2}\,du = 4\pi \left[\frac{2u^{3/2}}{3} - \frac{2u^{5/2}}{5}\right]_0^1$$

$$= 4\pi\left[\frac{2}{3} - \frac{2}{5}\right] = \pi\,\frac{16}{15}.$$

$\boxed{3}$ $\int_1^2 2\pi x^3 \sqrt{1+9x^4}\,d =$

$$\begin{array}{ll} u = 1+9x^4 & u(1) = 10 \\ du = 36x^3\,dx & u(2) = 145 \end{array}$$

$$S = \frac{2\pi}{36}\int_{10}^{145} \sqrt{u}\,du = \frac{2\pi}{36}\,\frac{2u^{\frac{3}{2}}}{3}\Big|_{10}^{145} = \frac{4\pi}{108}\left(145^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)$$

$$= \frac{\pi}{27}\left(\sqrt{145^3} - \sqrt{1000}\right).$$

$\boxed{4}$ $\int \coth^2 x\,\operatorname{csch}^2 x\,dx$

$$\begin{array}{l} u = \coth x \\ du = -\operatorname{csch}^2 x\,dx \end{array} \qquad = -\int u^2\,du = -\frac{u^3}{3} + C$$

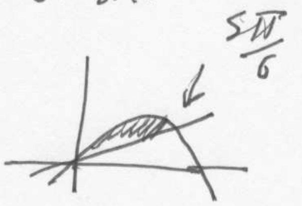
$$= -\frac{\coth^3 x}{3} + C.$$

[1] Eq. of the line $y - \cancel{\frac{1}{2}} = \frac{3}{5\pi}\left(x - \frac{5\pi}{6}\right) = \frac{3}{5\pi}x - \cancel{\frac{1}{2}} \Rightarrow y = \frac{3}{5\pi}x$

$y = \sin x$

Solving the two

$$\Rightarrow \sin x = \frac{3}{5\pi}x \Rightarrow x = \frac{5\pi}{6}$$



$$A = \int_0^{\frac{5\pi}{6}} \sin x - \frac{3}{5\pi}x \, dx = \frac{\sqrt{3}}{2} - \frac{5\pi}{24} + 1.$$

[2] disks $V = \pi\int_{-1}^{1}[1-y^2]^2 dy = \pi\int_{-1}^{1} 1 - 2y^2 + y^4 dy = \pi\left[y - \frac{2y^3}{3} + \frac{y^5}{5}\right]_{-1}^{1}$

$$= \pi\,\frac{16}{15}$$

Shells

$$V = 2\pi\int_0^1 x\left(\sqrt{1-x} - \left(-\sqrt{1-x}\right)\right) dx$$

$$= 2\pi\int_0^1 x\, 2\sqrt{1-x}\, dx = 4\pi\int_0^1 x\sqrt{1-x}\, dx \qquad u = 1-x,\quad du = -dx,\quad x = 1-u$$

$$= 4\pi\int_1^0 (1-u)\sqrt{u}\,(-du) = 4\pi\int_0^1 (1-u)\sqrt{u}\, du$$

$$= 4\pi\int_0^1 u^{1/2} - u^{3/2} du = \pi\,\frac{16}{15}$$

(3) $S = \int_1^4 2\pi\sqrt{x}\sqrt{1 + \frac{1}{4x}}\, dx = 2\pi\int_1^4 \sqrt{x + \frac{1}{4}}\, dx. = 2\pi\left.\frac{2\left(x+\frac{1}{4}\right)^{\frac{3}{2}}}{3}\right|_1^4$

$$= \frac{4\pi}{3}\left\{\left[4 + \frac{1}{4}\right]^{\frac{3}{2}} - \left[1 + \frac{1}{4}\right]^{\frac{3}{2}}\right\} =$$

(4) $\int \tanh^2 x \operatorname{sech}^2 x\, dx \qquad u = \tanh x$

$du = \operatorname{sech}^2 x\, dx$

$$\int u^2 du = \frac{u^3}{3} + C = \cancel{\tanh}$$

$$= \frac{\tanh^3 x}{3} + C.$$

Quiz #6 Math 102 012

Name:	I.D.	Sec.	

1 Find a formula for the general term of the sequance $r, -r^3, r^5, -r^7, \ldots$.

$$\left\{(-1)^n r^{2n+1}\right\}_{n=0}^{\infty}$$

2.Find the limit of the squance $\left\{\frac{n}{n+2}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{n}{n+2} = 1$$

3.Is the squance $\left\{\frac{2^n}{1+2^n}\right\}_{n=1}^{\infty}$ decrasing or increasing.

Increasing

$$\frac{a_{n+1}}{a_n} = \frac{\frac{2^{n+1}}{1+2^{n+1}}}{\frac{2^n}{1+2^n}} = \frac{2\cancel{2^n}}{1+22^n} \cdot \frac{1+2^n}{\cancel{2^n}} = \frac{2+22^n}{1+22^n} > 1$$

3.Is the serise $\sum_{n=1}^{\infty} \frac{4^{k-3}}{7^{k-1}}$ convergs or divergs if convergs find its sum.

G. S. with $r = \frac{4}{7}$ $a = \frac{1}{16}$

$$S = \frac{\frac{1}{16}}{1 - \frac{4}{7}}$$