King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 102
First Major

100

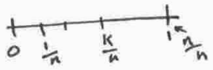
Name :------- ---ID # :-----------------------------------

Q1 (20 points) Find the Reimann integral by finding the limit of $\sum_{k=1}^{n} f(x_k^*)\Delta x$ for the function $f(x) = x^2 + x$ over the interval [0,1]. You may take regular partition.

The Reimann integral $\lim\limits_{\Delta x \to 0} \sum\limits_{k=1}^{n} f(x_k^*)\Delta x$

Now $\Delta x = \frac{1-0}{n} = \frac{1}{n}$; taking the rightend point $x_k^* = \frac{k}{n}$

Hence $f(x_k^*) = f(\frac{k}{n}) = (\frac{k}{n})^2 + \frac{k}{n}$



Therefor the Reimann integral $= \lim\limits_{n \to \infty} \left(\sum\limits_{k=1}^{n} (\frac{k}{n})^2 + \frac{k}{n} \right) \frac{1}{n}$

$\Delta x \to 0$
$\frac{1}{n} \to 0$
$n \to \infty$

$$= \lim_{n \to \infty} \frac{1}{n} \left\{ \frac{1}{n^2} \sum_{k=1}^{n} k^2 + \frac{1}{n} \sum_{k=1}^{n} k \right\}$$

$$= \lim_{n \to \infty} \frac{1}{n} \left\{ \frac{1}{n^2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{n} \frac{n(n+1)}{2} \right\}$$

$$= \lim_{n \to \infty} \frac{n(n+1)(2n+1) + \{3n\} n(n+1)}{6n^3}$$

$$= \lim_{n \to \infty} \frac{2n^3 + \cdots + 3n^3 + \cdots}{6n^3} = \frac{5}{6}.$$

Q2. (20 points) Solve the initial value problem $\frac{dy}{dx} = \sec^2 x - \sin x; \quad y(\frac{\pi}{4}) = 1$

$$y(x) = 1 + \int_{\frac{\pi}{4}}^{x} \sec^2 t - \sin t \, dt$$

$$= 1 + \int_{\frac{\pi}{4}}^{x} \sec^2 t \, dt - \int_{\frac{\pi}{4}}^{x} \sin t \, dt$$

$$= 1 + \tan t \Big|_{\frac{\pi}{4}}^{x} + \cos t \Big|_{\frac{\pi}{4}}^{x} = \cancel{1} + \tan x + \cos x - \tan\frac{\pi}{4} - \cos\frac{\pi}{4}$$

$$= \tan x + \cos x - \cos\frac{\pi}{4}$$

Q3. (15 points each; total 60 points) Evaluate

(a) $\frac{d}{dx}\int_{x^2}^{\ln x}\cos t\,dt = \cos(\ln x)\,\frac{1}{x} - \cos(x^2)\,2x$

$= \frac{1}{x}\cos(\ln x) - 2x\cos x^2.$

(b) $\int_0^{\frac{\pi}{6}} 2\cos 3x\,dx = \frac{2}{3}\int_0^{\frac{\pi}{2}} \cos u\,du$ let $u = 3x$ $du = 3\,dx$

$= \frac{2}{3}\sin u \Big|_0^{\frac{\pi}{2}}$

$= \frac{2}{3}\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{2}{3}.$

(c) $\int_0^{\frac{\pi}{12}} \frac{3^{\tan 3x}}{\cos^2 3x}\,dx = \int_0^{\frac{\pi}{12}} \frac{3}{3}(\sec^2 3x)\,3^{\tan 3x}\,dx$

let $u = \tan 3x$
$du = 3\sec^2 3x\,dx$

$= \frac{1}{3}\int_0^1 3^u\,du$

$= \frac{1}{3}\,\frac{3^u}{\ln 3}\Big|_0^1 = \frac{1}{3\ln 3}\left[3 - 3^0\right]$

$= \frac{2}{3\ln 3}$

(d) $\int \frac{e^x}{\pi^x}\,dx = \int \left(\frac{e}{\pi}\right)^x dx = \frac{\left(\frac{e}{\pi}\right)^x}{\ln\left(\frac{e}{\pi}\right)} + C$

$= \frac{e^x}{\pi^x(1 - \ln\pi)} + C$