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Q27/5.3 Consider the function $f(x) = x^{\frac{1}{3}}(5-x)$ Follow the steps to sketch the Graph of the function.

1) Find symmetry if any

No

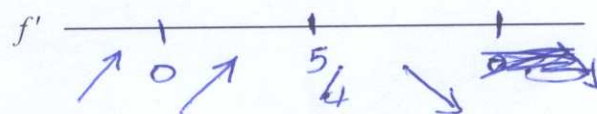
2) Find y-int. then x-int. then check if the graph above the x-axis or below.

(0,0) (0,5)



3) Find critical points then check if the graph increasing or decreasing, then find relative extreme

$$\begin{aligned}
 f'(x) &= \frac{1}{3} x^{-2/3} (5-x) - x^{1/3} \\
 &= \frac{5}{3} x^{-2/3} - \frac{1}{3} x^{1/3} - x^{1/3} \\
 &= \frac{5}{3} x^{-2/3} - \frac{4}{3} x^{1/3} = \frac{x^{-2/3}}{3} (5-4x)
 \end{aligned}$$

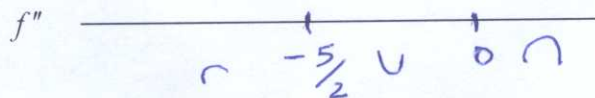


4) Find asymptotes if any

No

5) Check if the graph concave up or down then find inflection points if any

$$\begin{aligned}
 f''(x) &= -\frac{10}{9} x^{-5/3} - \frac{4}{9} x^{-2/3} \\
 &= -\frac{2}{9} x^{-5/3} (5+2x)
 \end{aligned}$$



- 6) Check the behavior of the graph as $x \rightarrow \infty$ and $x \rightarrow -\infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty = \lim_{x \rightarrow -\infty} f(x)$
- 7) Is there a cusp or a vertical tangent
- 8) Find the absolute extremum in the $[0,2]$
- 9) Sketch the graph

Ⓐ vertical tangent at $x=0$
 Since $\lim_{x \rightarrow 0^-} f'(x) = \infty = \lim_{x \rightarrow 0^+} f'(x)$

x	f(x)
0	Abs min
5/4	Abs max
2	

