

2. Find the limit of each of the following:

$$(a) \lim_{x \rightarrow 5} \frac{x^2 - 25}{|x - 5|}$$

$$(b) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$$

$$(c) \lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2}$$

$$(d) \lim_{x \rightarrow 0^-} \left[ \frac{1}{x} - \frac{1}{|x|} \right]$$

$$(e) \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$$

$$(f) \lim_{x \rightarrow 0} \frac{\sin^3 x}{x^3}$$

$$(g) \lim_{x \rightarrow 0^+} (1 + x)^{1/x}$$

$$(h) \lim_{x \rightarrow 1} \sin^{-1} \left( \frac{1 - \sqrt{x}}{1 - x} \right)$$

$$(i) \lim_{x \rightarrow \infty} \frac{e^{3x}}{x^4}$$

$$(j) \lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$$

$$(k) \lim_{x \rightarrow 0^+} (\cos x)^{1/x^2}$$

3. Find the derivative  $y'$  for each of the following:

$$(a) y = \sqrt{e^{2x} - \csc^3 x}$$

$$(b) y = \frac{\sec^2 5x^2 + 1}{1 + \cos^{-1} x}$$

$$(c) xy = \cot(xy)$$

$$(d) yx + 1 = 3 \tan^{-1} y$$

$$(e) y = \sin(\tan \sqrt{\sin x})$$

$$(f) y = 10^{\sin x^2}$$

4. If  $x^4 + y^4 = 16$ , show that  $y'' = -48 \frac{x^2}{y^7}$ .
5. Suppose  $f$  is a one-to-one differentiable function and its inverse  $f^{-1}$  also differentiable. Use implicit differentiation to show that

$$\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}, \quad \text{where } f' \neq 0.$$

6. Use the definition of derivative to show that  $\frac{d}{dx} [\log_b x] = \frac{1}{x \ln b}$ ,  $x > 0$ .
7. show that the equation  $4x^3 - 6x^2 + 3x - 2 = 0$  has a real root between 1 and 2.
8. Show that the function  $f(x) = |x - 3|$  is continuous everywhere.
9. Given

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ e^x & \text{if } x < 0. \end{cases}$$

Discuss the continuity of  $f$  at  $x = 0$ .

10. Find the horizontal and vertical asymptotes of the graph of  $f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$ .
11. Find the critical points of  $f(x) = 4x^{3/5} - x^{8/5}$ .
12. Find the absolute max and absolute min of  $f(x) = x^4 - 2x^2 + 3$  on  $[-2, 3]$ .
13. Sketch the graph of  $\frac{2x - 5}{x + 3}$ .
14. State Rolle's theorem and verify that the function  $f(x) = \sin 2\pi x$  satisfies the hypotheses of Rolle's theorem on the interval  $[-1, 1]$ . Then find a number  $c$  that satisfies its conclusion on this interval.
15. Is it true that the equation  $y = y''' + 5y' - 6$  is satisfied by  $y = x$ ?
16. Is it true that the inverse function of  $y = \sin x$  is  $y = \frac{1}{\sin x}$ ?
17. Is it true that the function  $y = \ln x$  is differentiable everywhere?
18. Is it true that if  $k(x) = f(g(x))$ , then  $\frac{d^2 k}{dx^2} = f'(g) \cdot g'' + f''(g) \cdot (g')^2$ ?
19. What is the error in the following steps:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos x}{2x} = \lim_{x \rightarrow 0} \frac{-\sin x}{2} = 0$$

and determine the correct value of this limit.

20. Use local linear approximation to approximate  $\sin 29^\circ$ .