Quiz #3 Math101 021

Name:	I.D.		

36/3.4 Find $\frac{d^{17}}{dx^{17}}(x\sin x)$

$17 = 4(4)+1$

$f' = \sin x + x\cos x$ $\quad \longrightarrow \quad f^{(17)} = 17\sin x + x\cos x$

$f'' = \cos x + \cos x - x\sin x$

$= 2\cos x - x\sin x$

$f''' = -3\sin x - x\cos x$

$f^{(4)} = \cancel{+4\sin x} - 4\cos x + x\sin x$

30/3.6 Find equations of all tangent lines to the graph of $y^3 + yx^2 + x^2 - 3y^2 = 0$ at $x = 0$

at $x=0$ $\quad y^3 + 0 + 0 - 3y^2 = 0$

$y^2(y-3) = 0 \quad y = 0$

or $y = 3$

$3y^2y' + y'x^2 + 2yx + 2x - 6yy' = 0$

$y'[3y^2 + x^2 - 6y] = -2yx - 2x$

$y' = \frac{-2yx - 2x}{3y^2 + x^2 - 6y}$ $\qquad y'\Big|_{(0,0)} \Rightarrow$ undefine V. t. at $x = 0$

$y'\Big|_{(0,3)} = \frac{0}{27-18} = 0$ $\quad$ H. t. at $\underline{y=3}$

Quiz #4 Math101 021 Key

Name:	I.D.		

23/3.8 Use local linear approximation to approximate $\sqrt{80.9}$

$x_0 = 81$

$x = 80.9$

$\Delta x = -0.1$

$f(x_0) = \sqrt{81} = 9$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{18}$

$f(x) = \sqrt{x} \qquad f(x) \simeq f(x_0) + f'(x_0)(x - x_0)$

$$\sqrt{80.9} = f(x) \simeq 9 + \frac{1}{18}(-0.1)$$

$$= 9 - \frac{0.1}{18}$$

$$= 9 - \frac{1}{180} = \frac{1621}{180}$$

28/4.1 Prove that $f(x) = 3x^2 + 5x - 2$ is one to one in the restricted domain $x \geq 0$ then find the inverse in that domain (Hint: complete the squrae).

$f'(x) = 6X + 5 \geq 0 \quad \forall x \geq 0$ then $f(x)$ is increasing in the domain

$\Rightarrow f(x)$ 1-1 $\Rightarrow f(x)$ has an inverse

$$Y = 3x^2 + 5x - 2 = 3(x^2 + \tfrac{5}{2}x) - 2$$

$$Y = 3(x^2 + \tfrac{5}{3}x + \tfrac{25}{36}) - 2 - \frac{3(25)}{36}$$

$$Y = 3(X + \tfrac{5}{6})^2 - \cancel{\frac{147}{36}} - \frac{49}{12}$$

$$\sqrt{\frac{Y}{3} + \frac{49}{36}} = X + \frac{5}{6}$$

$$\frac{Y + \frac{147}{36}}{3} = (x + \tfrac{5}{6})^2$$

$$\cancel{\frac{Y + 147}{36}} = (x + \tfrac{5}{6})^2$$

$$x = -\frac{5}{6} + \sqrt{\frac{36Y + 147}{108}} \Rightarrow f^{-1}(x) = -\frac{5}{6} + \sqrt{\frac{36x + 147}{108}}$$

Quiz #5 Math 101, 021

Name:	I.D.		

Q28/5.2 Consider the function $f(x) = x^4 + 2x^3$

Find:

1) Relative extreme

$$f'(x) = 4x^3 + 6x^2 = 2x^2(2x+3)$$

$-$ $+$ $+$

$-\frac{3}{2}$ 0

Max	min	none
	$-3/2$	0

2) Increasing or decreasing

Intervals of increasing	Intervals of decreasing
$[-3/2, \infty)$	$(-\infty, -3/2]$

3) Inflection points

$$f''(x) = 12x^2 + 12x = 12x(x+1)$$

$+$ $-$ $+$

$\cup$ -1 $\cap$ 0 $\cup$

IP
$0, -1$

4) Concave up or down

Intervals of Concave up	Intervals of Concave down
$(-\infty, -1), (0, \infty)$	$(-1, 0)$

Quiz #6 Math 101 Semester 021

Name:	I.D.		

Q27/5.3 Consider the function $f(x) = 2x + 3x^{\frac{2}{3}}$ Follow the steps to sketch the Graph of the function.

1) Find symmetry if any

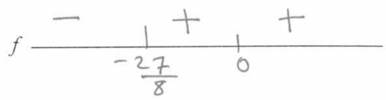
No

2) Find y-int. then x-int. then check if the graph above the x-axis or below.

(0,0)

$x^{\frac{2}{3}}(2x^{1/3}+3)=0$

$x=0$ or $x^{1/3} = -\frac{3}{2} \Rightarrow x = -\frac{27}{8}$

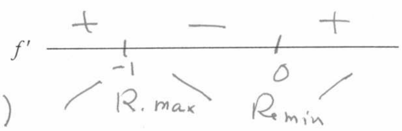


3) Find critical points then check if the graph increasing or decreasing , then find relative extreme

$f'(x) = 2 + 2x^{-\frac{1}{3}} = 0$

$x=0$ c.p f' not define

$x=-1$ C.P $f'=0$ ($x^{-1/3}=-1$)



4) Find asymptotes if any

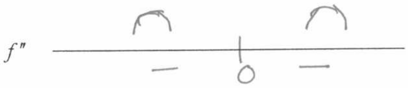
No V.A.

N. H.A.

5) Check if the graph concave up or down then find inflection points if any

$f''(x) = -\frac{2}{3}x^{-4/3}$

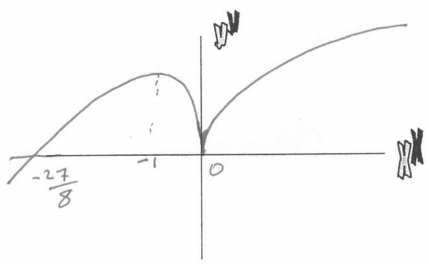
$f''=0$ $x=0$



6) Check the behavior of the graph as $x \to \infty$ and $x \to -\infty$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$



$\lim_{x \to 0^-} f'(x) = -\infty$

$\lim_{x \to 0^+} f'(x) = \infty$

Quiz #7 Math 101 Semester 021

Name:	I.D.		

Q2/3.2 Given that the tangent line to $y = f(x)$ at a point (-1,3) passas through the point (0,4), find $f'(-1)$

$f'(-1) =$ slope of the tangent line $= \frac{3-4}{-1-0} = 1$

Q8/4.5 find the limit $\lim\limits_{t \to 0} \frac{te^t}{1-e^t}$ $\left(\frac{0}{0}\right) = \lim\limits_{t \to 0} \frac{te^t + e^t}{-e^t} = \frac{0+1}{-1} = -1$

Q8/5.5 Find the Abs. Max and Abs min. $f(x) = 2x^3 - 3x^2 - 12x$ in the interval [-2,3]

$f'(x) = 6x^2 - 6x - 12 = 6[x^2 - x - 2] = 6(x-2)(x+1)$

C.P. are $x = 2$, $x = 1$

endpoints are $x = -2$, $x = 3$

$f(-2) = -4$

$f(-1) = 7$

$f(2) = -20$

$f(3) = -9$

then $x = 2$ is the Abs min.

$x = -1$ is the Abs max.