Q1 When proving that $\lim_{x\to 9}\sqrt{x} = 3$ what is the maximum value of δ when $\varepsilon = 0.001$?(15pts)

Using definition 2.3.3

$\sqrt{x} \in (3-\varepsilon, 3+\varepsilon)$ *or* $x \in (9-6\varepsilon+\varepsilon^2, 3+6\varepsilon+\varepsilon^2) = (9-\delta_1, 9+\delta_2)$

Hence $\delta = \min(\delta_1, \delta_2)$ *=0.00599 is the maximum value*

Q2. Prove using δ -ε definition $\lim_{x\to 3}(x^2-5) = 4$? (15pts)

Using difference between two squares gives $|x-3| < \varepsilon/5$ *since*

$\frac{1}{|x+3|} < 1/5$ *. Thus the value of* $\delta \le \varepsilon/5$

Q3. Evaluate the following limits if they exist. But if they do not exist, give reasons.

a. $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$ =1/4

b. $\lim_{x\to 0^-}\frac{x}{\sqrt{x^2}}$ =-1

c. $\lim_{x\to 2}\frac{\cos(\pi/x)}{x-2} = \pi$

d. $\lim_{x\to 0}\frac{2x+\sin 3x}{x}$ =5

Q4. If $0 \le f(x) \le a$ for some real number a , prove that $\lim_{x\to 0}|x| f(x) = 0$ (10pts)

Squeezing theorem

Q5. Using Intermediate value theorem show that the equation $x^3 - 4x + 1 = 0$ has a solution between 1 and 2?

Since f(1)=- while f(2)=+

Q6. $f(x)\begin{cases} 2x-3 & x<2 \\ c & x=2 \\ x^2 & x>2 \end{cases}$

a. Find $\lim_{x\to 2^+} f(x)$ =4

Find $\lim_{x\to 2^-} f(x)$ =1

b. Is there a value of c which makes *f(x)* continuous ?

No