Quiz #2 Math 101 Semester 041

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1. Find values for the constant k and h that will make the following function continuous

$$f(x)=\begin{cases} k+3x^2 & x<1 \\ h & x=1 \\ 2k+x & x>1 \end{cases}$$

$f(x)$ continuous if $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\Rightarrow \lim_{x\to 1^-} k+3x^2 = \lim_{x\to 1^+} 2k+x$

$\Rightarrow k+3 = 2k+1$

$\Rightarrow 3-1 = 2k-k$

$\boxed{k=2}$

$\lim_{x\to 1} f(x) = 5 = f(1)$

$\Rightarrow \boxed{h=5}$

2. Find $\lim_{h\to 0} \dfrac{2-2\cos^2 3h}{h\tan h}$

$$\lim_{h\to 0} \frac{2(1-\cos^2 3h)}{h \tan h} = \lim_{h\to 0} \frac{2 \sin^2 3h}{h \sin h} \cdot \cos h$$

$$= 2 \lim_{h\to 0} \frac{\sin^2 3h}{h(h)} \cdot \frac{h}{\sin h} \cdot \lim_{h\to 0} \cos h$$

$$= 2 \lim_{h\to 0} 9\underbrace{\left(\frac{\sin 3h}{3h}\right)^2}_{=1} \underbrace{\lim_{h\to 0} \frac{h}{\sin h}}_{=1} \cdot \underbrace{\lim_{h\to 0} \cosh}_{=1}$$

$$= 2(9) = 18$$

3 . Find $\lim_{x\to 1}\dfrac{\sin(x^2-3x+2)}{x^3-1}$

$$= \lim_{x\to 1} \frac{\sin(x^2-3x+2)}{x^2-3x+2} \cdot \frac{x^2-3x+2}{x^3-1}$$

$$\boxed{\text{If } x\to 1 \quad x^2-3x+2 \to 0}$$

$$= \lim_{x^2-3x+2\to 0} \frac{\sin(x^2-3x+2)}{x^2-3x+2} \cdot \lim_{x\to 1} \frac{(x-1)(x-2)}{(x-1)(x^2+x+1)}$$

$$= 1 \cdot \frac{-1}{3} = -\frac{1}{3}$$

4. Find $\lim_{x\to -\infty}\dfrac{\sqrt{3x^2-x+2}}{x-3}$

$$= \lim_{x\to -\infty} \frac{\sqrt{3x^2-x+2}/|x|}{x-3/|x|}$$

$$= \lim_{x\to -\infty} \frac{\sqrt{\frac{3x^2}{x^2} - \frac{x}{x^2} + \frac{2}{x^2}}}{\frac{x}{|x|} - \frac{3}{|x|}} = \frac{\sqrt{3-0+0}}{-1-0}$$

$$= -\sqrt{3}.$$