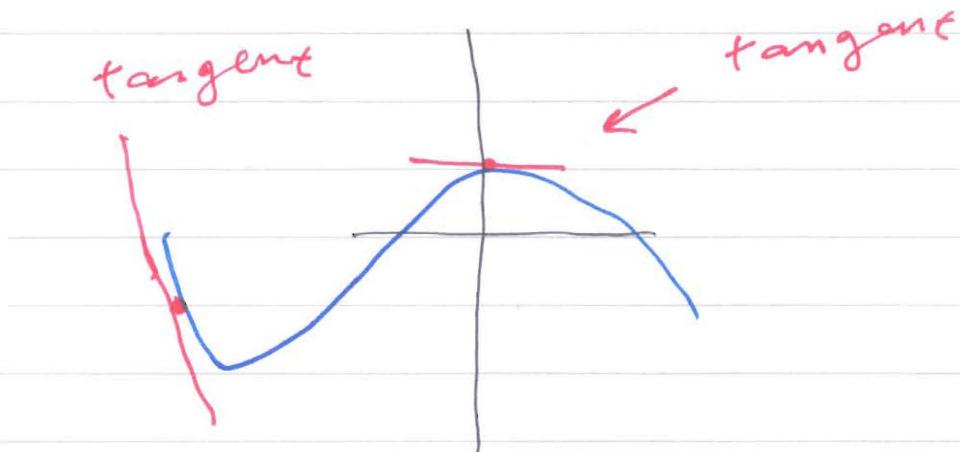


①

## 2.1 The tangent and velocity problems

how functions changes

tangent means touching



Geometrically

The slope of the line through  
the points  $P(x_1, f(x_1))$  &  $Q(x_2, f(x_2))$

..

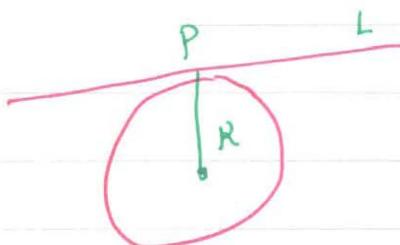
(2)

Tangent line & the slope of tangent line

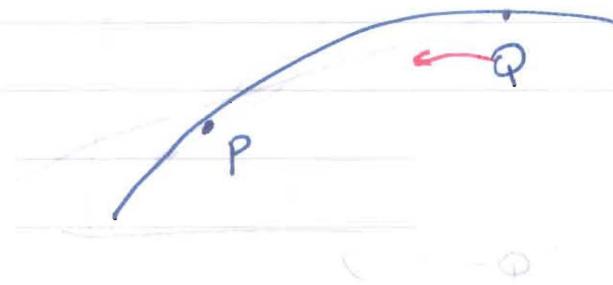
It is the line L touches the curve at point P

Ex.

If the curve is circle then the tangent line L at P and R the radius passes through P then ~~R & L~~ are perpendicular to L



For the slope



Start with Q & P the secant line  
let Q get closer & closer to P at  
the limit along the curve we have the tangent line

If the limit exist (slope of line)  
then it define the tangent line

(3)

$$\text{Ex. } f(x) = x^3 \quad [-1, 0]$$

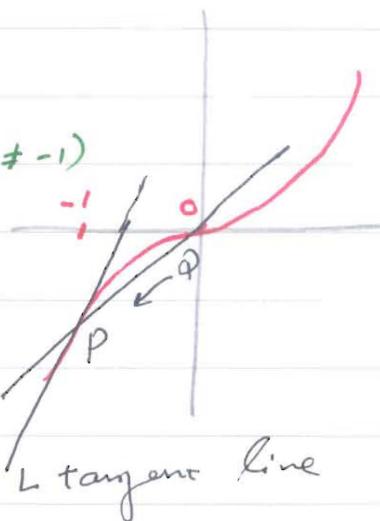
The slope of the secant line

$$M_{PQ} = \frac{\Delta y}{\Delta x} = \frac{(-1)^3 - 0}{-1 - 0} = 1$$

$$\frac{\Delta y}{\Delta x} = \frac{(x+h)^3 - x^3}{h}$$

Now let  $x \rightarrow -1$  (but  $x \neq -1$ )

$$h \rightarrow 0$$



If we choose  $x = -\frac{1}{4}, \dots, -0.99$

$$M_{PQ} = \frac{\frac{1}{8} - (-1)}{-\frac{1}{2} - (-1)} = \frac{7/4}{-1/2} = -7/4$$

$$x: -0.15 \quad -0.05 \quad -0.9 \quad -0.99 \quad -0.999$$

$$x^3: \frac{0.0156}{-0.0125} \quad \frac{-0.0156}{0.0125} \quad -0.729 \quad -0.97 \quad -0.997$$

$$M_{PQ}: 1.3125 \quad 1.25 \quad 2.71 \quad 2.97 \quad 2.997$$

$M_{PQ} \rightarrow 3$  when  $x \rightarrow -1$  &  $Q \rightarrow P$ .

or  $\lim_{x \rightarrow -1} \frac{x^3 - (-1)}{x - (-1)} = 3$  the slope of the tangent line is 3.

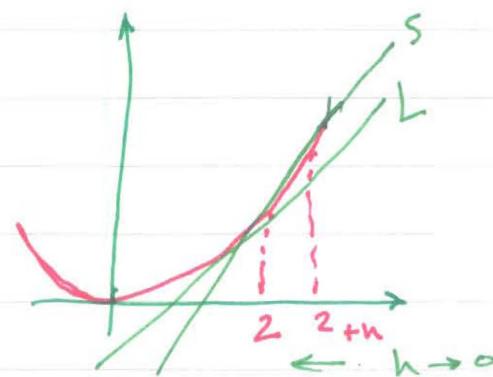
~ ~

(4)

~~Ex.~~  $y = x^2$   $P(2, 4)$   $Q(2+h, (2+h)^2)$

$$\text{Secant slope } \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(2+h)^2 - 2^2}{h} = \frac{h^2 + 4h + 4 - 4}{h} = h+4$$



$$\Delta y = 2^2 - x^2 = (2+h)^2 - 4$$

$$h \rightarrow 0 \quad x \rightarrow 2 \quad \lim_{\substack{\Delta x \rightarrow 0 \\ h \rightarrow 0 \\ x \rightarrow 2}} \frac{\Delta y}{\Delta x} = 4.$$

So that limit called slope of the tangent line  
or Instantaneous Rate of change.  
or velocity

which is the speed at exact point of time