3.(a) / 38

$$\tan^{-1}x = \sum_{i=1}^{n} (-1)^{i+1} \frac{x^{2i-1}}{2i-1} + R_{n+1}.$$

$$\tan^{-1}1 = \frac{\pi}{4}.$$

$$|4P_n(1) - \pi| < 10^{-3} \Rightarrow \left|P_n(1) - \frac{\pi}{4}\right| < \frac{10^{-3}}{4}.$$

where $P_n(1) \approx \tan^{-1}1$ with error $R_{n+1}^{(1)}$.

Now, $R_{n+1}^{(x)} = \dfrac{(-1)^{n+2} x^{2n+1}}{2n+1}$

Put $\left|R_{n+1}^{(1)}\right| = \left|\dfrac{1}{2n+1}\right| < \dfrac{10^{-3}}{4}.$

$\Rightarrow \quad 2n+1 > 4000 \quad \Rightarrow n \geq 2000$

7.(a) / 38

$$\lim_{h\to 0} \frac{\sinh - h\cosh}{h} = 0.$$

Expand sin and cos in h about $x_0 = 0$

$$\lim_{h\to 0} \left[\left(h - \frac{h^3}{3!} + \cdots\right) - h\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} \cdots\right)\right] / h. = 0$$

$\therefore$ convergence is $O(h^2)$

$$\Rightarrow \lim_{h\to 0} O(h^2) = 0$$

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$$\left(\cdots\left(\left(\left(a_n x + a_{n-1}\right)x + a_{n-2}\right)x + \cdots\right)x + a_0 = P(x).\right.$$

Algorithm: input $x_0, a_n, a_{n-1}, \ldots, a_0$.

output $P(x_0)$.

set $P(x_0) = a_n x_0 + a_{n-1}$.

for $i = 2, n$.

set $P(x_0) = P(x_0)\, x_0 + a_{n-i}$

stop.

Q3

input $x_0, \underline{a}, n$

output p

Set $p = a_n$

For $i = n-1 \dots 0$

$p = p x_0 + a_i$

(6) multiplications $\sum_{j=1}^{n} \sum_{i=1}^{j} 1 = \sum_{j=1}^{n} j = \frac{n(n+1)}{2}$.

addition is same.

(5) n multiplications and ~~same~~ additions