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3.9 Related Rates

In many applications there are variables x and y satisfies some relationship over some interval of time t .

We know rate of change of one & we need to find the other
the problem called Related rates

$$x = f(t) \quad y = g(t)$$

Ex relationship $x^3 - 2y^2 + 5x^2 = 16$

We know $\dot{x} = \frac{dx}{dt} = 4$, $x = 2$, $y = -1$

Find $\frac{dy}{dt} = \dot{y}$

$$3x^2\dot{x} - 2(2y)\dot{y} + 5(2x\dot{x}) = 0$$

$$\dot{y} = \frac{10x\dot{x} + 3x^2\dot{x}}{4y} = \frac{10(2)(4) + 3(4)(4)}{-4}$$

We need to solve real world problems

Ex. A ladder 20 ft long leans against the wall of a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec how fast is the ladder sliding down the building when the top of the ladder is 12 ft above the ground

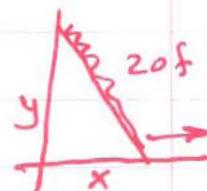
Ans x = distance from bl to bottom of the ladder
 y = $\sqrt{x^2 + y^2}$ = ground - top

$$\dot{x} = \frac{dx}{dt} = 2 \quad \text{when } y = 12 \text{ ft}$$

$$x^2 + y^2 = 20^2 \quad \leftarrow x = 16$$

$$2x\dot{x} + 2y\dot{y} = 0$$

$$\dot{y} = \frac{dy}{dt} = -\frac{x}{y} \dot{x} = -\frac{16}{12} \cdot 2 = -\frac{8}{3} \text{ ft/sec.}$$



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Guidelines for Solving Related Rates Problems.

- 1 * Read the problem carefully many times (Unknown, quantity)
- 2 * Sketch a picture (Figure) diagram and label variables.
- 3 * Write down all the known facts; ~~Find~~^{write} the known rate of change and identify the unknown rate of change.
- 4 * Find an equation relating the quantity.
- 5 * Differentiate the eq. and then solve for the unknown rate of change
- 6 * Substitute the known values and rates.

Ex. At 1:00 pm ship A is 25 mi. due south of ship B.

If ship A is sailing west at a rate of 16 mi/hr and ship B is sailing south at a rate of 20 mi/hr find the rate of change at which the distance between the ships is changing at 1:30 P.M.

Ans.

① Read the problem

$$\begin{aligned} \textcircled{3} \quad & \frac{dy}{dt} = 20 \quad \frac{dx}{dt} = 16 \\ & \text{Find } \frac{d^2z}{dt^2}. \quad t = \frac{1}{2} \end{aligned}$$

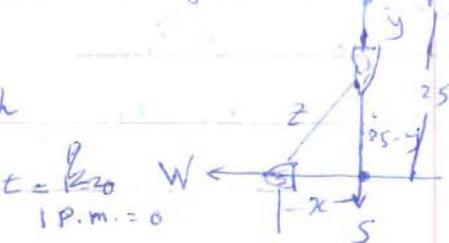
$$\textcircled{4} \quad z^2 + (25-y)^2 = z^2$$

1:30 P.M. the ships traveled for half an hour

$$x = \frac{1}{2}(16) = 8 \quad y = \frac{1}{2}(20) = 10 \quad \text{and } 25-y=15$$

$$\text{then } z = 17$$

② Sketch



$$\frac{dz}{dt} = -\frac{172}{17} \text{ mi/hr}$$

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Ex A water tank has the shape of an inverted right circular cone of altitude 12 ft. and base radius 6 ft. If water is being pumped into the tank at a rate of 10 gal/min, approximate the rate at which the water level is rising when the water is 3 ft. deep. ($1 \text{ gal} \approx 0.1337 \text{ ft}^3$)

Ans.

r and h are the radius of the surface and the depth and both function of time.



$$\frac{dV}{dt} = 10 \text{ gal/min}$$

$$\text{Find } \frac{dh}{dt} \text{ when } h=3.$$

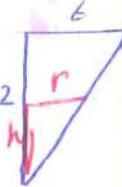
Now - Volume

$$V = \frac{1}{3}\pi r^2 h.$$

before differentiating let us express V in terms of one variable.

Similar triangles

$$\frac{6}{12} = \frac{r}{h} \Rightarrow r = \frac{h}{2}$$



$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{4}{\pi(9)} (1.337) \approx 0.189 \text{ ft/min.}$$

Ex At a certain instant, the radius of a circle is 5 cm. and increasing at the rate of 2 cm/sec. How fast is the area increasing at that instance.

Ans. $A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(5)2 = 20\pi \text{ cm}^2/\text{sec.}$