

### 3.4 The chain Rule

How to find the derivative  $f \circ g$

Ex.  $y = \sin(3x)$

$$y' = \cos(3x) \cdot \underline{\underline{3}}$$

$$y = f \circ g = \sin(g(x)) \quad g(x) = 3x$$

$$y = f(g(x))$$

$$\frac{dy}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

Th° If  $f(u)$  is diff. at  $u=g(x)$  and  $g(x)$  is diff at  $x$ , then the composite  $f \circ g(x) = f(g(x))$  is differentiable at  $x$

$$(f \circ g)'(x) = f'(g(x)) g'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex.

①  $f(u) = (3u^2 + 2u - 1)^6$

$$y' = 6(3u^2 + 2u - 1)^5 \cdot (6u + 2)$$

②  $f(u) = \sin u^3$

$$f'(u) = \cos u^3 \cdot 3u^2$$

③  $f(u) = \sec(u)$

$$f' = \sec u \tan u \cdot \frac{du}{dx}$$

(65)

- (10) Find A so that  $y = As \sin 3t$  satisfies  
the equation  $\frac{d^2y}{dx^2} + 2y = 4 \sin 3t$ .

$$y' = 3A \cos 3t \quad y'' = -9A \sin 3t$$

$$-9A \sin 3t + 2A \sin 3t = 4 \sin 3t$$

$$-9A + 2A = 4$$

$$A = -4/7.$$

- (11)  $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x=0 \end{cases}$  find  $f'$  if it exists

$f(x)$  cont at  $x=0$  but not diff at  $x=0$

Using squeeze th<sup>n</sup>  $f'(0) = \lim_{h \rightarrow 0} \frac{h \sin \frac{1}{h} - 0}{h}$

$$x \neq 0 \quad f'(x) = \sin \frac{1}{x} + x \left(-\frac{1}{x^2}\right) \cos \frac{1}{x} \quad \text{DNE}$$

- (12) Given  $f'(0)=2, g(0)=0, g'(0)=3$

$$\text{find } (f \circ g)'(0) = f'(g(0))g'(0) = 6$$

$$\frac{d}{dx} a^x = \ln a \quad a^x$$

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}, \quad \frac{d[g(x)]^n}{dx} = n [g(x)]^{n-1} \cdot \frac{dg(x)}{dx}.$$

(67)

④  $y = \sin^3 x$

$$f(u) = u^3 \quad u = \sin x$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = 3u^2 \cdot \cos x$$

⑤  $f(x) = \cos^2(3\sqrt{x})$

$$f' = 2\cos(3\sqrt{x}) \cdot (-\sin 3\sqrt{x}) \cdot \frac{3}{2\sqrt{x}}$$

⑥  $f(x) = \sqrt{\csc x}$

$$f' = \frac{1}{2} (\csc x)^{-\frac{1}{2}} \cdot 5$$

⑦  $f(x) = \sqrt{x} + \tan^3 \sqrt{x}$

$$f' = \frac{1}{2\sqrt{x}} \tan^3 \sqrt{x} + \sqrt{x} \cdot 3 \tan^2 \sqrt{x} \cdot \sec^2 \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

⑧  $f(x) = (1 + \sin^3(x^5))^{12}$

$$f' = 12(1 + \sin^3(x^5))^{11} \cdot (3 \sin^2 x^5 \cdot \cos x^5 \cdot 5x^4)$$

⑨ Find the eq. of tangent line of  $y = \sec^3(\frac{\pi}{2} - x)$   
at  $x = -\frac{\pi}{2}$ .

$$y' = -3 \sec^2(\frac{\pi}{2} - x) \cdot \sec(\frac{\pi}{2} - x) \tan(\frac{\pi}{2} - x)$$

$$y'(-\frac{\pi}{2}) = 0 \quad y(-\frac{\pi}{2}) = -1$$

$y = -1$