3.3 Derivative of trigonometric functions.

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

Ex. $\lim_{x \to 0} \frac{\sin 2x}{5x} = \lim_{x \to 0} \frac{2}{5} \frac{\sin 2x}{2x} = 2/5.$

Ex $\lim_{x \to 2} \frac{\cos \pi/x}{x-2} \left(\frac{0}{0}\right) = \lim_{t \to 0} \frac{\cos \frac{\pi}{2} - t}{\frac{4t}{\pi - 2t}}$ put $t = \frac{\pi}{2} - \frac{\pi}{x}$ then $x \to 2 \Rightarrow t \to 0$

$$= \lim_{t \to 0} \frac{1}{4} \frac{\sin t}{t} \cdot \lim_{t \to 0} (\pi - 2t)$$

$$= \pi/4.$$

or $\lim_{x \to 2} \frac{\sin \frac{\pi}{2} - \frac{\pi}{x}}{x-2} = \lim_{x-2 \to 0} \frac{\pi}{2x} \frac{\sin \frac{\pi}{2x}(x-2)}{x-2} = \pi/4.$

Ex. $\lim_{x \to 0} \frac{2x + \sin 3x}{x} = 5.$ Ex. $\lim_{x \to a} \frac{x^2 - a}{\sin x + a} = 2a.$

Ex. $\lim_{t \to 0} \frac{\cos^2 t - 2\cos t + 1}{t^2} = \lim_{t \to 0} \frac{(\cos t - 1)^2}{t^2}$

$$= \left(\lim_{t \to 0} \frac{\cos t - 1}{t}\right)^2$$

$$= 0.$$

$\frac{d}{dx} \sin x = \cos x$ $\quad$ $\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$ $\quad$ $\frac{d}{dx} \cot x = -\csc^2 x$

$\frac{d}{dx} \sec x = \sec x \tan x$ $\quad$ $\frac{d}{dx} \csc x = -\csc x \cot x$.

Ex.

① $y = \tan x$ prove $y' = \sec^2 x$

② $y = \sec x \tan x$

$$\frac{dy}{dx} = \sec x \tan^2 x + \sec^3 x$$

③ $f(x) = 2\cos x - 3\sin x$

$f'(x) =$

④ $y(\theta) = \frac{\csc\theta}{\tan\theta}$ $\quad$ $y' = \frac{-\csc\theta \cot\theta \tan\theta - \csc\theta \sec^2\theta}{\tan^2\theta}$

$$= \frac{-\csc\theta(1+\sec^2\theta)}{\tan^2\theta}$$

⑤ Find the equation of the line tangent to the graph of $\cos x$ at the point $x = \pi/2$.

$x = \pi/2 \quad y = 0$

$m = -\sin \pi/2 = -1$

Eq $\rightarrow$ $y = -x + \pi/2$

⑥ $y = \frac{\tan x}{1+\tan x}$ $\quad$ $y' = \frac{\sec^2 x}{(1+\tan x)^2}$

7 Find $\dfrac{d^{23}}{d\,x^{23}}(x \sin x)$

Ans $y = x \sin x$

y'

y''

$f(x) = \sin(3x - \cancel{bt})$

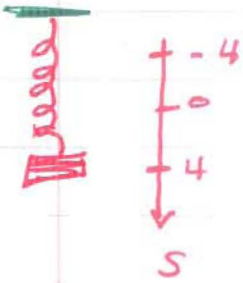
$f^{(x)} = \begin{cases} (-1)^k 3^k \sin & k \text{ odd} \\ (-1)^k 3^k \cos & k \text{ odd} \end{cases}$

Nex section
Chain Rule

Patrron

8 An object at the end of a vertical spring is stretched 4 cm beyond its rest position and released at time $t=0$

Find the velocity at time t and use it to analyze the motion of the object



The position fuction

$S(t) = 4 \cos t.$

Ans

$$v = \frac{ds}{dt} = -4 \sin t$$

The object ocsillates between -4 & 4, the period of oscillation is 2π ($\cos t$)

The speed $|v| = 4|\sin t|$

max when $|\sin t| = 1$ $t = \pi/2, -\pi/2$

fastest (max velocity) $t=0, s=0$

speed $= 0$, $\sin t = 0$ $t = 0$