MATH 102
QUIZ # 6

NAME: .... SEC. #: . . . . . .

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Q1. Find the Taylor polynomial of order 4 for $\sin x$ about $\frac{\pi}{2}$.

$$P_4(x) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x-\frac{\pi}{2}) + \frac{f''(\frac{\pi}{2})}{2!}(x-\frac{\pi}{2})^2 + \frac{f'''(\frac{\pi}{2})}{3!}(x-\frac{\pi}{2})^3 + \frac{f^{(4)}(\frac{\pi}{2})}{4!}(x-\frac{\pi}{2})^4$$

$$P_4(x) = 1 - \frac{(x-\frac{\pi}{2})^2}{2} + \frac{(x-\frac{\pi}{2})^4}{24}$$

Q2. Use any method to show that

$$\left\{\frac{n^2}{n!}\right\}_{n=1}^{+\infty}$$

is eventually strictly increasing or eventually strictly decreasing.

$$a_{n+1} = \frac{(n+1)^2}{(n+1)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{n^2} < 1 \text{ for } n \geq 2$$

eventually strictly decreasing

Q3. Determine whether the sequence in **Q2** converges, and if so find its limit.

Converges, since $M = 0$ is a lower bound.

$$a_{n+1} = \frac{n+1}{n(n-1)!} = \frac{a_n}{n} + \frac{1}{n!}$$

As $n \to +\infty$, $a_n \to L$ and $a_{n+1} \to L$

$L = 0$