

MATH 102
QUIZ # 4

NAME: SEC. #:

ID #:

Q1. Prove the identity:

$$\cosh^2 x + \sinh^2 x = \cosh 2x$$

$$\begin{aligned} \text{L.H.S.} &= \left(\frac{e^x + e^{-x}}{2} \right)^2 + \left(\frac{e^x - e^{-x}}{2} \right)^2 \\ &= \frac{1}{4} [(e^{2x} + 2e^0 + e^{-2x}) + (e^{2x} - 2e^0 + e^{-2x})] \\ &= \frac{1}{4} (2e^{2x} + 2e^{-2x}) = \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = \text{R.H.S.} \end{aligned}$$

Q2. Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis.

See example 2 page 494

Q3. Find the arc length of the curve $y = \cosh x$ between $x = 0$ and $x = \ln 2$

$$\frac{dy}{dx} = \sinh x$$

$$\begin{aligned} L &= \int_0^{\ln 2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx \\ &= \int_0^{\ln 2} \sqrt{1 + \sinh^2 x} dx \\ &= \int_0^{\ln 2} \cosh x dx = \sinh x \Big|_0^{\ln 2} \\ &= \frac{e^{\ln 2} - e^{-\ln 2}}{2} = \frac{3}{4} \end{aligned}$$