Mathematics and Real Life: Reflections on Teaching Mathematics

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What is the use of it after all? Where is it used in real life? These questions have become a refrain and a bane in the modern class room- at least in the affluent and consumer societies.

Mathematicians a hundred years ago would have had no difficulty in giving convincing answers- for many of them were involved in interdisciplinary research. For historical reasons, Mathematics had to undergo a period of intense introspection from which it is only now emerging. Most of us have thus grown up doing mathematics for its own sake, because we enjoy doing it and it satisfies our basic desire to know. We feel secure in the knowledge that our 'useless' activities do turn out to be of use eventually. If confronted seriously, we take recourse to history, with illustrious predecessors like Apollonius, Hadamard and Ramanujan- to name a few. We have read that mathematics is used in breaking codes and building mammoth structures like the modern ships and aero planes. Their construction goes beyond our normal experiences and mathematics becomes a necessity. Yet, we cannot make a truly convincing case to the skeptical student because our authority is wanting. Our lack of practical knowledge, of knowledge of other subjects, is indeed a real hindrance.

Fortunately, most mathematicians have a natural empathy with the arts and a very convincing case can be made about the usefulness of mathematics in arts and of the arts themselves. The internet is a wonderful resource for this. For example, if you search for the mathematics of perspective drawing, you will find a wealth of beautiful and brilliant articles on this theme. Particularly illuminating are the articles inspired by Marc Frantz's Mathematics and Arts [1].

We, as teachers, could emphasize in our class rooms the close connection of arts with sciences. Here, the visual arts are the most convincing. The artist discovered the rules of representing three dimensions in two and the mathematician/artist discovered the precise rules which make the electronic reproduction of such visual information possible.

In the Islamic world, we could make students conscious of the contributions of, say, Ibn-Al-Haytham (Al Hazen) [2], whose researches in the theory of vision are the foundation of perspective drawing, thereby making for them an emotional connection with modern science and its spirit of enquiry.

So how does a computer represent three dimensions on a flat screen?

It does this more or less in the same way as an artist renders a three dimensional impression on a canvas. The key to this are the rules of perspective drawing discovered by artists and their subsequent translation and implementation using software languages.
What are these rules? Briefly, these are:

- Given a view-point P_0, lines which are parallel to the plane of the canvas are projected from P_0 on the canvas to parallel lines.
- Lines which are parallel, but not parallel to the plane of the canvas, project from P_0 to a family of lines, all of which converge to a point V- called the vanishing point.
- There is a precise formula for the image of the projections using, as a coordinate system an orthogonal frame on the canvas and a line perpendicular to the canvas and passing through the view-point P_0. This formula involves the viewing distance from the canvas.

These rules can be assigned as exercises, involving nothing more than vectors and their scalar and vector products, which are taught at the sophomore level. Here is a more formal description of these rules:

- Taking the plane of the canvas as the (X, Y) plane and the view-point P_0 with coordinates (0, 0,-d), the line joining the point $P(x, y, z)$ with P_0 intersects the (X, Y) plane at the point $(xd/(z+d), yd/(z+d))$: this point is the projection of P on the canvas.
- Lines with direction vectors parallel to the vector $[a,b,c]$ project to a pencil of lines which all pass through the point $(ad/c, bd/c)$: this point is the *vanishing*

point for the representation of parallel lines onto the canvas; it is the key to the creation of depth in painting.

- Lines with direction vector $[a,b,0]$ project to lines on the plane of the canvas with the same direction vector.

This circle of ideas also opens up the possibility of investigating higher dimensions and gaining a glimpse of 4 dimensional objects.

Another chapter of the undergraduate curriculum which has direct impact on contemporary life is the theory of conic sections. The properties of conic sections and surfaces of revolutions obtained from them have well-known and everywhere present applications: their focusing properties are used for the transport of signals and energy. These applications are so much within our everyday experiences that a few words are enough for appreciating the role of mathematics in life.

The references given below discuss in great detail the aesthetics and history of these ideas, and serve also as teaching resources for traditional art and computer graphics.

The reader who has done the exercises given above will see that they are within the capabilities of sophomore students. Their accessibility leads to a deep appreciation of mathematics and its role in the arts and sciences. One feels connected to so many things. It is an emotional experience.

REFERENCES

[1] Marc Frantz, Drawing with Awareness, http://www.mathaware.org/mam/03/essay6.html

[2] J. J. O'Connor and E. F. Robertson: Mathematics and Art, http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Art.html

[3] Cathi Sanders, The Geometry of 3-D Drawing, http://mathforum.org/workshops/sum98/participants/sanders/