Q1. (3+1= 4 Points) A College of Business wants to install a copy machine to be used by staff. From experience at other colleges, the dean believes the number of documents is normally distributed with a daily standard deviation of 44 copies. The machine is tested for 5 days and the resulting daily mean is 345 copies.

  a. Give a 92% confidence interval for the mean number of pages copied.
  b. Suppose the dean will install the copier if he is confident that the daily average number of copies exceeds 290. Does the results of a) justify purchasing a copier? Explain.

$n = 5, \quad \bar{x} = 345 \quad \sigma = 44 \qquad \alpha = 1 - .92 = .08 \quad \frac{\alpha}{2} = .04$

$|Z_{.04}| = 1.75$

Assume the population is normal

(a) A 92% Confidence interval is

$\bar{x} \mp Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} ; \quad 345 \mp 1.75 \frac{44}{\sqrt{5}} ; \quad 345 \mp 34.435$

$310.565 \leq \mu \leq 379.435$

(b) Yes, since 290 is less than the lower bound = 310.45.

Q2. (4+2=6 Marks) Ali is interested in estimating the average purchase amount for convenience stores in the city of Dammam. He selected a random sample of 12 purchases from several convenience stores and get the following data set

13	4	9	17	27	11	9	16	30	24	14	19

You are given the standard error of the sample mean to be **2.5**.

  a. Construct and interpret a 90% confidence interval for the population mean amount of purchases. State your **assumptions**.
  b. If Ali wishes to estimate the population mean to be within ± 3 with 95% confidence, what is the sample size he needs to insure these conditions?

$\bar{x} = 16.083 ; \quad S_{\bar{x}} = \frac{S}{\sqrt{n}} = 2.25 \quad \Rightarrow \quad S = 2.25\sqrt{12} = 7.794$

$\alpha = 1 - .9 = .1, \quad \frac{\alpha}{2} = .05 \quad df = 12 - 1 = 11 \qquad t_{.05, 11} = 1.7959$

Assume Normal Population

(a) A 90% Confidence interval for μ is

$\bar{x} \mp t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} ; \quad 16.083 \mp 1.7959(2.25)$

$12.0392 \leq \mu \leq 20.13074$

(b) $n = \frac{Z_{\frac{\alpha}{2}}^2 \, \sigma^2}{e^2} = \frac{(1.96)^2 (7.794)^2}{(3)^2} = 25.93 = 26$

Q3. (3+3=6 Marks) There are 2320 students in a small university. Currently, classrooms are segregated. To cut on cost the University Management is considering offering non segregated classes at senior level. A survey of 340 students yields 124 students who favor no segregation in classrooms.

a. Develop a 95% confidence interval for the proportion of students who favor no segregation.

b. If the management wish to estimate the population proportion to be within ∓ 0.04 with 98% confidence level, determine the sample size that they need.

$$nP = 124 \quad n(1-P) = (340-124) = 216 > 5 \checkmark$$

$$P = \frac{124}{340} = 0.365$$

(a) A 95% confidence interval for the proportion π is

$$P \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{P(1-P)}{n}} \; ; \; .365 \mp 1.96 \sqrt{\frac{.365(1-.365)}{340}}$$

$$.365 \mp 0.051 \Rightarrow \boxed{0.341 \le \pi \le 0.416}$$

(b) $\quad \alpha = 1-.98 = .02, \quad \frac{\alpha}{2} = .01 \quad |Z_{.01}| = 2.331$

$$n = \frac{Z_{\frac{\alpha}{2}}^2 \; \pi(1-\pi)}{e^2} = \frac{(2.33)^2 (.365)(1-.365)}{(0.04)^2}$$

$$n = 787$$

Q4. (3+1= 4 Marks) The new manager of sport club would like to know how long current members have been members of the club. He selects a sample of 45 current members. The mean length of membership of the sample is 6.38 years and the sample standard deviation is 1.85 years.

a. Construct a 90 % confidence interval for the population mean.

b. The former manager reported a mean of about 7.5 years. Does the sample information support this claim? Explain

$$n = 45, \bar{X} = 6.38, S = 1.85$$

Since $n = 45 > 30$ we can use the Z-Table to find $Z_{.05}$

$$\alpha = 1-.9 = .10, \quad \frac{\alpha}{2} = .1/2 = .05 \quad |Z_{.05}| = 1.645$$

(a) A 90% Confidence for the Population mean is

$$\bar{X} \mp Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} \; ; \; 6.38 \mp 1.645 \frac{1.85}{\sqrt{45}}$$

$$6.38 \mp 0.454 \; , \; \boxed{5.926 \le \mu \le 6.834}$$

(b) No, since 7.5 is not inside the Confidence interval

$$(5.92, 6.834)$$