1 1.3 Quadratic Equations

1. A **quadratic equation** in x is an equation that can be written in the form $ax^2 + bx + c$, where $a \neq 0$.

2. If the quadratic polynomial in a quadratic equation is factorable over the set of integers, then the equation can be solved by factoring and using the **zero product property**.

3. If the quadratic equation can be written in the form $(ax + b)^2 = c$, then the equation can be solved by **taking square property**.

4. Every quadratic equation can be solved by **completing the square** or by using **the quadratic formula**.

5. The quadratic formula, for the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

6. The quadratic equation $ax^2 + bx + c = 0$, with real coefficients and $a \neq 0$, has **discriminant** $b^2 - 4ac$.

   (a) If $b^2 - 4ac > 0$, then the quadratic equation has two distinct real roots.

   (b) If $b^2 - 4ac = 0$, then the quadratic equation has a real root that is a double root.

   (c) If $b^2 - 4ac < 0$, then the quadratic equation has two distinct complex roots that are not real. The roots are conjugates of each other.

2 1.4 Other Types of Equations

1. If the polynomial in an equation is factorable over the set of integers, then the equation can be solved by factoring and using the **zero product property**.

2. If the equation involves radical expression, then we can use the **Power Principle**.

3. The power principle states that, if P and Q are algebraic expressions and n is a positive integer, then every solution of $P = Q$ is a solution of $P^n = Q^n$.

4. Any solution of $P^n = Q^n$ that is not a solution of $P = Q$ is called an **extraneous solution**. Extraneous solutions maybe introduced whenever we raise each side of an equation to an even power.

5. An equation is said to be **quadratic in form** if it can be written in the form $au^2 + bu + c = 0$, where $a \neq 0$ and u is an algebraic expression.

3 1.5 Inequalities

1. The set of all solutions of an inequality is he solution set of the inequality.
2. **Equivalent inequalities** have the same solution set.
3. **Adding** the same real number to each side of an inequality **preserves** the direction of the inequality symbol.
4. **Multiplying** each side of an inequality by the same **positive** real number **preserves** the direction of the inequality symbol.
5. **Multiplying** each side of an inequality by the same **negative** real number **changes** the direction of the inequality symbol.
6. To solve an inequality, use the properties of an inequality or **the critical value method**.
7. A **compound inequality** is formed by joining two inequalities with the connective word and or or.
8. The solution set of a compound inequality with the connective word **or** is the **union** of the solution set of the two inequalities.
9. The solution set of a compound inequality with the connective word **and** is the **intersection** of the solution set of the two inequalities.
10. For any variable expression E and any nonnegative real number k,
    (a) $|E| \leq k$ if and only if $-k \leq E \leq k$.
    (b) $|E| \geq k$ if and only if $E \leq -k$ or $E \geq k$
11. Nonzero polynomials in x have the property that for any value of x between two consecutive real zeros, either all values of the polynomial are positive or all values of the polynomial are negative.
12. The **critical values of the polynomial inequality** are the real zeros of the polynomial.
13. The **critical values of a rational expression** are the numbers that cause the numerator of the rational expression to equal zero or the denominator of the rational expression to equal zero.

4 2.1Two-Dimensional Coordinate System and Graphs

1. The **distance** d between the points represented by (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

2. The **Midpoint** of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

3. If $(x_1, 0)$ satisfies an equation, then the point $(x_1, 0)$ is called an **x-intercept** of the graph of the equation.

4. If $(0, y_1)$ satisfies an equation, then the point $(0, y_1)$ is called an **y-intercept** of the graph of the equation.

5. The **standard form of the equation of a circle** with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

5 2.2 Introduction to Functions

1. A **function** is a set of ordered pairs in which no two ordered paries that have the same first coordinate have different second coordinate.

2. Unless otherwise stated, the **domain** of a function is the set of all real numbers for which the function makes sense and yields real numbers.

3. If a and b are elements of an interval I that is a subset of the domain of a function f, then

   (a) f is **increasing** on I if $f(a) < f(b)$ whenever $a < b$.

   (b) f is **decreasing** on I if $f(a) > f(b)$ whenever $a < b$.

   (c) f is **constant** on I if $f(a) = f(b)$ whenever $a < b$.

4. A graph is **the graph of a function** if and only if no *vertical line* intersects the graph at more than one point.

5. If every *horizontal line* intersects the graph of a function at most once, then the graph is the graph of a **one-to-one function.**

6. The Greatest Integer Function.

6 2.3 linear Functions

1. A function is a **linear function** of x if it can be written in the form $f(x) = mx + b$, where m and b are real numbers and $m \neq 0$.

2. The **slope** m of the line passing through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ with $x_1 \neq x_2$ is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

3. The graph of the equation $f(x) = mx + b$ has slope m and y-intercept $(0, b)$.

4. The graph of $y - y_1 = m(x - x_1)$ is a line that has slope m and passes through (x_1, y_1).

5. Let l_1 be the graph of $f_1(x) = m_1x + b$ and Let l_2 be the graph of $f_2(x) = m_2x + b$. Then

   (a) l_1 and l_2 are **parallel** if and only if $m_1 = m_{2.}$

   (b) l_1 and l_2 are **perpendicular** if and only if $m_1 = -\frac{1}{m_2}$.

7 2.4 Quadratic Functions

1. A **quadratic function** of x is a function that can be represented by an equation of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

2. Every quadratic function $f(x) = ax^2 + bx + c$ can be written in the **standard form** $f(x) = a(x-h)^2 + k$, $a \neq 0$.

   (a) The graph of a parabola with **vertex** (h, k).

   (b) The parabola is symmetric with respect to the vertical line $x = h$, which is called the **axis of symmetry**.

   (c) If $a > 0$, then

   (d) the parabola **opens up**

   (e) the vertex is the **lowest point** on the graph of the parabola

   (f) the y-coordinate k of the vertex is the **minimum** value of the function

   (g) the **range** is $\{y|y \geq k\}$.

   (h) If $a > 0$, then

   (i) the parabola **opens down**

   (j) the vertex is the **highest point** on the graph of the parabola

   (k) the y-coordinate k of the vertex is the **maximum** value of the function

   (l) the **range** is $\{y|y \leq k\}$.

3. The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

8 2.5 Properties of Graphs

1. The graph of an equation is **symmetric** with respect to

   (a) **the y-axis** if the replacement of x with $-x$ leaves the equation unaltered.

   (b) **the x-axis** if the replacement of y with $-y$ leaves the equation unaltered.

   (c) **the origin** if the replacement of x with $-x$ and y with $-y$ leaves the equation unaltered.

2. The function f is an **even function** if $f(-x) = f(x)$ for all x in the domain of f.

3. The function f is an **odd function** if $f(-x) = -f(x)$ for all x in the domain of f.

4. If f is a function and c is a positive constant, then

   (a) $y = f(x) + c$ is the graph of $y = f(x)$ shifted **up vertically** c units.

   (b) $y = f(x) - c$ is the graph of $y = f(x)$ shifted **down vertically** c units.

   (c) $y = f(x + c)$ is the graph of $y = f(x)$ shifted **left horizontally** c units.

   (d) $y = f(x - c)$ is the graph of $y = f(x)$ shifted **right horizontally** c units.

5. The graph of

   (a) $y = -f(x)$ is the graph of $y = f(x)$ **reflected across the x-axis**.

   (b) $y = f(-x)$ is the graph of $y = f(x)$ **reflected across the y-axis**.

6. (a) If $0 < c < 1$, then the graph of $y = c \cdot f(x)$ is obtained by **shrinking** the graph of $y = f(x)$ **vertically**.

   (b) If $c > 1$, then the graph of $y = c \cdot f(x)$ is obtained by **stretching** the graph of $y = f(x)$ **vertically**.

7. (a) If $a > 1$, then the graph of $y = f(ax)$ is **a horizontal shrinking** of the graph of $y = f(x)$.

   (b) If $0 < a < 1$, then the graph of $y = f(ax)$ is **a horizontal stretching** of the graph of $y = f(x)$.