1 P6 Rational Expressions

A **rational expression** is a fraction in which the numerator and the denominator are polynomials. $\frac{3}{x+1}$ and $\frac{x^2-4x-21}{x^2-9}$ are rational expressions.

The **Domain** of a rational expression is the set of all real numbers that can be used as replacements for the variable where division by zero is excluded from the domain.

Example 1 *Find the domain of* $\frac{7x}{x^2-5x}$.

Properties of Rational Expressions $\frac{P}{Q}$ and $\frac{R}{S}$ where $Q \neq 0$ and $S \neq 0$.

Equality $\frac{P}{Q} = \frac{R}{S}$ if and only if $PS = RQ$. Equivalent Expressions $\frac{P}{Q} = \frac{PR}{QR}$, $R \neq 0$. Sign $-\frac{P}{Q} = \frac{-P}{Q} = \frac{P}{-Q}$.

Simplify a rational expression

1) Factor the numerator and the denominator.

2) Use equivalent expressions property to eliminate factors common to both.

Example 2 *Simplify* 1) $\frac{7+20x-3x^2}{2x^2-11x-21}$ 2) $\frac{x^2-7x+12}{x^2-9x+20}$

Operations on Rational Expressions

For all rational expressions $\frac{P}{Q}$, $\frac{R}{Q}$, and $\frac{R}{S}$ where $Q \neq 0$ and $S \neq 0$.

Addition $\frac{P}{Q}+\frac{R}{Q} = \frac{P+R}{Q}$ Subtraction $\frac{P}{Q}-\frac{R}{Q} = \frac{P-R}{Q}$ Multiplication $\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$ Division $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$, $R \neq 0$.

Example 3 *Simplify* 1) $\frac{x^2-3x+2}{x^2+x-6} \div \frac{x^3+x^2-2x}{x^2+5x+6}$ 2) $\left(\frac{y^3+4y^2-5y}{y^2-2y+1} \div \frac{y^2+y-2}{y^4+8y}\right) \cdot \frac{y-1}{y^2-2y+4}$ 3) $\frac{5x}{18}+\frac{x}{18}$

Determining the Least Common Denominator (LCD) of rational expression

1)Factor each denominator completely and express repeated factors using exponential notation.

2)Identify the largest power of each factor in any single factorization.

LCD is the product of each factor raised to its largest power.

Example 4 *Find the LCD of* 1) $\frac{1}{x+3}+\frac{5}{2x-1}$. 2) $\frac{5x}{(x+5)(x-7)^3}-\frac{7}{x(x+5)^2(x-7)}$ 3) $\frac{x+y}{x^2+2xy-3y^2}-\frac{2y}{x^2+xy-6y^2}+\frac{5}{3(x-y)}$

Example 5 *Perform the indicated operation:* 1) $\frac{5x}{48}+\frac{x}{15}$ 2) $\frac{x}{x^2-4}-\frac{2x-1}{x^2-3x-10}$

Complex Fractions

A complex fraction is a fraction whose numerator and denominator contains one or more fractions.

Methods for simplifying Complex Fractions

Method 1 Multiply by the LCD

1)Determine the LCD of all the fractions in the complex fraction.

2)Multiply both the numerator and the denominator by the LCD.

3) If possible, simplify the resulting rational expression.

Method 2 Multiply by the reciprocal of the denominator

1)Simplify the numerator to a single fraction and the denominator to a single fraction.

2)Multiply the numerator by the reciprocal of the denominator.

3) If possible, simplify the resulting rational expression.

Example 6 *Simplify* 1) $\frac{\frac{2}{x-2}+\frac{1}{x}}{\frac{3x}{x-5}-\frac{2}{x-5}}$ 2) $\frac{c^{-1}}{a^{-1}+b^{-1}}$ 3) $\frac{\frac{x^2+3x-10}{(x+3)(x-2)}}{\frac{(x+5)(x+6)}{2x^2-15x+18}}$ 4) $\frac{1-(1-x)^{-1}}{x^{-1}+(1-x)^{-1}}$

5) $\frac{-2x^3-18}{x^2-64} \div \frac{x^3-3x^2+9x-27}{x^2+5x-24}$ 6) $\frac{3+\frac{2}{1-\frac{3}{x}}}{4+\frac{1}{2+\frac{1}{x}}}$ 7) $\left(\frac{a^{-1}b-ab^{-1}}{a^2-b^2}\right)^{-1}$

8) $\left(x-1-\frac{6}{x}\right) \div \left(1+\frac{2}{x}-\frac{15}{x^2}\right)$ 9) $\frac{x+y}{x-y} \cdot \frac{x^{-1}-y^{-1}}{x^{-1}+y^{-1}}$