1 P5 Factoring

Factoring means writing a polynomial as a product of polynomials of lowest degree.

We will consider only he factorization of polynomials that have integer coefficients and we will deal with factoring over the integers.

1)**Greatest Common Factor (GCF)**

Given two or more exponential expressions with the same prime base or the same variable base, the GCF is the exponential expression with the smallest exponent.

Example 1 *Find the GCF of* 1)2^3, 2^5, *and* 2^8. 2)a^4 *and* a. 3)$27a^3b^4$ *and* $18b^3c$. 4)$3x\left(2x+5\right)$ *and* $4\left(2x+5\right)^2$

Example 2 *Factor out the GCF:* 1)$10x^3+6x$. 2)$-6x^2y^2+3xy^2$ 3)$15x^{2n}+9xn^{n+1}-3x^n$. 4)$2x\left(3x+1\right)-\left(3x+1\right)$

2)**Factoring Trinomials**

Some trinomials of the form x^2+bx+c can be factored by **a trial procedure**.

Points to remember to factor x^2+bx+c as $\left(x+a_1\right)\left(x+a_2\right)$

1) The constant term c is the product a_1a_2

2) The coefficient b is a_1+a_2.

3) If $c>0$, then a_1, a_2 have the same sign as b.

4) If $c<0$, then a_1, a_2 have opposite sign.

Example 3 *Factor:* 1)$x^2+7x-18$. 2)$x^2+8x+15$. 3)$x^2-9x+14$.

Points to remember to factor ax^2+bx+c, $a>0$.

1) If $c>0$, then a_1, a_2 have the same sign as b.

2) If $c<0$, then a_1, a_2 have opposite sign.

3) If the terms of the trinomial do not have common factor, then neither binomial will have a common factor.

4) ax^2+bx+c can be factored as the product of two binomials with integer coefficients if and only if b^2-4ac is a perfect square.

Example 4 *Determine whether each trinomial is factorable over the integers*

1)$4x^2+8x-7$ 2)$6x^2-5x-4$.

Example 5 *Factor* 1)$6x^2-11x+4$ 2)$4x^2-17x-21$

Some polynomials of degree greater than 2 can be factored by the trial method.

Example 6 *Factor* 1)$6x^6-11x^3+4$ 2)$6x^{2n}-11x^n+4$ 3)$6x^2-11xy+4y^2$ 4)$6(x+y)^2-11(x+y)z+4z^2$.

Special Factoring

Some polynomials can be factored by making use of the following factoring formulas:

1)**Difference of two squares** $x^2 - y^2 = (x - y)(x + y)$

2)**Perfect-square trinomials** $x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

3)**Sum of two cubes** $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$

4)**Difference of two cubes** $x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$

Example 7 *Factor* 1)$49x^2 - 144$ 2)$x^2 + 4x + 4$ 3)$16m^2 - 40mn + 25n^2$ 4)$8a^3 + b^3$ 5)$a^3 - 64$ 6)$27x^6 - (x - y)^3$

7)$x^{4n} - 1$ 8)$4y^4 - 5y^2 + 1$ 9) $(x - y - 2z)^2 - (2x + y - z)^2$

The trinomial $ax^2 + bx + c$ is a perfect square if $b^2 - 4ac = 0$.

Example 8 *Find k such that the trinomial is a perfect-square trinomial.* 1)$x^2 + 16x + k$ 2)$x^2 + kx + 16$

Factor by Grouping

Pairs of terms that have a common factor are first grouped together.

Example 9 *Factor:* 1)$6y^3 - 21y^2 - 4y + 14$ 2)$a^2 + 10ab + 25b^2 - c^2$. 3)$p^2 + p - q - q^2$.

General Factoring Strategy

1)Factor out the GCF of all terms.

2)Try to factor a binomial as a)difference of two squares b)sum or difference of two cubes

3)Try to factor a trinomial a)as a perfect square trinomial b) using the trial method.

4)Try to factor a polynomial with more than three terms by grouping.

5)After each factorization, examine the new factors to see whether they can be factored.

Example 10 *Factor:* 1)$x^6 + 7x^3 - 8$ 2)$-5a^4b - 5a^3b^2 + 30a^2b^3$ 3)$3ab^2 + 9a - 2ab^3 - 6ab$ 4)$3x^2 + xy - 2y^2 - x - y$

5)$6\left(4x^2 - 12xy + 9y^2\right) + 7(2x - 3y) - 3$ 6)$x^2y - xy^2 + x^3 - y^3$ 7)$15y - 3x - 10y^2 + 2xy$ 8)$y^4 + 64$ 9)$2 + 4x - 10x^4 - 5x^3$

Example 11 *Show that* $2x$ *is a factor of* $x^3 - 3x^2y + 3xy^2 - y^3 + (x + y)^3$.