1 P4 Polynomials

Monomial is a constant, a variable, or a product of a constant and one or more variables, with the variables having only nonnegative integer exponents. The constant is called the **coefficient** of the monomial. The **degree** of the monomial is the sum of the exponents of the variables.

Example 1 *Find the coefficient and the degree of the monomial* $-5xy^2$.

Example 2 *Is* $3x^{-2}$ *a monomial?*

The **Polynomial** is the sum of finite number of monomials. Each monomial is called a term of the polynomial. The degree of a polynomial is the largest degree of the terms in the polynomial.

Example 3 *Determine the degree of the polynomial* $-9x^5y + 10xy^4 - 11x^2y^2$.

Like terms are the terms that have exactly the same variables raised to the same powers.

Example 4 *Are* $14x^2y$ *and* $-31x^2y$ *like terms?*

Example 5 *Are* $2x^3y$ *and* $7xy$ *like terms?*

A Polynomial is simplified if all its like terms has been combined.

Example 6 *Simplify* $4x^2 + 3x + 5x$

A simplified polynomial with two terms is called a **binomial**. $(4x + 7)$
A simplified polynomial with three terms is called a **trinomial**. $(2x^3 - 7x^2 + 11)$
A nonzero constant is called a **constant** Polynomial and its degree is zero. $(5 = 5x^0)$
0 is defined to be a Polynomial with no degree.
The **General Form** of a polynomial of degree n in the variable x is $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where $a_n \neq 0$ and n is a nonnegative integer. We call a_n the **leading coefficient** and a_0 the **constant term**.
A polynomial is in its **standard form** if it is written with decreasing powers of x.

Example 7 *Write the following Polynomial in standard form* $3x^2 - 4x^3 + 7x^4 - 1$.

Example 8 *Which one of the following is a polynomial?*   1)$x^2 + 3x + 2x^{-1}$ 2)$\frac{x^3+4}{x-7}$ 3)$3\left(\frac{1}{x}\right)^2$
4)$5x^4 + 3x + 7$ 5)$2x + \sqrt{x}$

Example 9 *Find the leading coefficient, degree, terms, and coefficients for the following polynomials*   1)$9x^2 - x + 5$ 2)$5x + x^3 - 3$

Example 10 *Let $P(x) = 3x^4 - 6x^2 + 2x^5 + 7x^3 - x + 10$. Then only one of the following is TRUE:* 1)$P(x) + \frac{1}{x}$ *is a polynomial.* 2)*the degree of $P(x)$ is* 6. 3)*There are 5 terms in $P(x)$.* 4)$P(x)$ *is in simplest form.* 5)*The leading coefficient of $P(x)$ is equal to 2.*

Addition and Subtraction To add polynomial, we add like terms

Example 11 *Simplify $(3x^2 + 7x - 5) + (4x^3 - 2x + 1)$*

Example 12 *Find the additive inverse of the polynomial $3x - 7$.*

Example 13 *Simplify $(2x - 5) - (3x - 7)$*

Product

Example 14 *Simplify $(3x - 4)(2x^2 + 5x + 1)$*

We use the distributive property to find the product of the polynomials.
Another method is to do it like multiplying the numbers
We can use **Foil method** to do the product of $(a + b)(c + d)$

Example 15 *Simplify $(7x - 2)(5x + 4)$*

Special Product Formulas
(Sum)(Difference) $(x - y)(x + y) = x^2 - y^2$.
(Binomial)^2 $(x + y)^2 = x^2 + 2xy + y^2$
    $(x - y)^2 = x^2 - 2xy + y^2$
(Binomial)^3 $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
    $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - x^3$

Example 16 *Perform the indicated operation:* 1) $(7x + 10)(7x - 10)$
2) $[(x - 2y) + 7][(x - 2y) - 7]$ 3) $(2y^2 + 11z)^2$ 4) $(4d - 1)^2 - (2d - 3)^2$ 5) $(3x + 2y)^3$

Example 17 *Find the coefficient of a^2b in the product $(a - 2b)^2(3a - b)$*

Example 18 *Find the coefficient of a^{2x} in the product $(a^x - 2)^3(a^x + 1)^2$*

Example 19 *Find the sum of the coefficients of all terms in the expression $(2x - 3y)^3$*

To evaluate a polynomial, substitute the given value(s) for the variable(s) and then use the order of operations agreement.
    Order of Operations Agreement
    If grouping symbols are present, evaluate by performing the operations within the grouping symbols, innermost grouping symbols first, while observing the order given in steps 1 to 3.
    1)Evaluate each power.
    2)Do all multiplications and divisions working from left to right.
    3)Do all additions and subtractions working from left to right.

Example 20 *Evaluate $2x^3 - 6x^2 + 7$ for $x = -4$.*