1 P2 Intervals, Absolute Value, and Distance

The real numbers can be represented geometrically by a coordinate axis called a **real number line**. The number associated with a particular point on a real number line is called the **coordinate** of the point. The point corresponding to zero is called **the origin**, denoted by 0. Numbers to the right of the origin are positive real numbers; numbers to the left of the origin are negative real numbers.

There is a one-to-one correspondence between the real numbers and the points on the real number line and that means each real number corresponds to one and only one point on the real number line and vice versa.

A certain order relationships exist between real numbers. For example, if a and b are real numbers, then

a equals b (denoted by $a = b$) if $a - b = 0$ and that means the point with coordinate a is the same point as the point with coordinate b.

a is greater than b (denoted by $a > b$) if $a - b$ is positive and that means the point with coordinate a is to the right of the point with coordinate b.

a is less than b (denoted by $a < b$) if $a - b$ is negative and that means the point with coordinate a is to the left of the point with coordinate b.

We can combine the inequality symbols $<$ and $>$ with the equality symbol in the following manner:

$a \geq b$ a is greater than or equal to b which means $a > b$ or $a = b$.

$a \leq b$ a is less than or equal to b which means $a < b$ or $a = b$.

Inequalities can be used to represent subsets of the real numbers.

Example 1 1)$x > 2$ *represents all real numbers greater then* 2.

Example 2 2)$x \leq 1$ *represents all real numbers less than or equal to* 1.

Example 3 3) $-1 \leq x < 3$ *represents all real numbers between* -1 *and* 3, *including* -1 *but not* 3.

Interval Notation

Subsets of real numbers can be represented by a compact form of notation called interval notation. $[-1, 3]$ is the interval notation for the last example.

In general, the interval notation

(a, b) represents all real numbers between a and b, not including a and not including b. This is an open interval. Using inequalities, this is written $a < x < b$ and the graph is

$[a, b]$ represents all real numbers between a and b, including a and including b. This is a closed interval. Using inequalities, this is written $a \leq x \leq b$ and the graph is

$(a, b]$ represents all real numbers between a and b, not including a and including b. This is a half-open interval. Using inequalities, this is written $a < x \leq b$ and the graph is

$[a, b)$ represents all real numbers between a and b, including a and not including b. This is a half-open interval. Using inequalities, this is written $a \leq x < b$ and the graph is

Subsets of the real numbers whose graphs extend forever in one or both directions can be represented by interval notation using the infinity symbol ∞ or the negative infinity symbol $-\infty$.

$(-\infty, a)$ represents all real numbers less than a. Using inequalities, this is written $x < a$ and the graph is

(b, ∞) represents all real numbers greater than b. Using inequalities, this is written $x > b$ and the graph is

$(-\infty, a]$ represents all real numbers less than or equal to a. Using inequalities, this is written $x \leq a$ and the graph is

$[b, \infty)$ represents all real numbers greater than or equal to b. Using inequalities, this is written $x \geq b$ and the graph is

$(-\infty, \infty)$ represents all real numbers. Using inequalities, this is written $-\infty < x < \infty$ and the graph is

Some graphs consist of more than one interval of the real number line. The word *or* is used to denote the union of the sets. the word *and* is used to denote the intersection of the sets.

Example 4 *Write the following using inequality notation and then graph them*

1) $(-\infty, 2)$ 2) $[-4, -2] \cup (0, \infty)$ 3) $[-2, 4] \cap (3, 6)$ 4) $[1, 3) \cap (3, 5]$ 5) $(-\infty, 6) \cap (3, \infty)$ 6) $(-\infty, 4) \cup (2, \infty)$

Example 5 *Write the following using interval notation and then graph them*

1) $-2 \leq x < 3$ 2) $x < -2$ or $x \geq 1$ 3) $x > 3$ and $x \leq 4$ 4) $x > -4$ or $x < 1$ 5) $x \geq 4$

Absolute Value

The absolute value of the real number a, denoted $|a|$, is the distance between a and 0 on the number line. The absolute value of the real number a is defined by $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

Example 6 *Write each expression without absolute value symbols.*

1) $|4|$ 2) $|-8|$ 3) $-|-3| - |8|$ 4) $|-x^2 - 1|$ 5) $|-1 + \pi|$

Example 7 *If $A = \pi - 3$, then find the value of $|A| + |-A| - |-3 - A|$.*

Absolute Value Theorems

For all real numbers a and b,

Nonnegative $|a| \geq 0$ Product $|ab| = |a||b|$ Quotient $|\frac{a}{b}| = \frac{|a|}{|b|}, b \neq 0$ Triangle Inequality $|a+b| \leq |a|+|b|$ Difference $|a-b| = |b-a|$

Example 8 *Given* $-3 < x < 0$, *write* $|\frac{x}{|x|+|x+3|}|$*without using absolute value symbols.*

Example 9 *Given* $4 \leq x \leq 5$, *write* $|x-3| + |x-6|$ *without using absolute value symbols.*

Distance between the points on a number line

For any real numbers a and b, the distance between the graph of a and the graph of b is denoted by $d\left(a,b\right)$, where $d\left(a,b\right) = |a-b|$

Example 10 *Find the distance between the points whose coordinates are* -4 *and* -7 *on the real number line.*

Example 11 *Express "The distance between a real number* x *and* 7 *is less than* 2*" using absolute value notation.*

Example 12 *Express "The distance between a real number* x *and* 0 *is greater than or equal to* 1*" using absolute value notation.*

Example 13 *Express "x is more than b units from a but less than c units from a" using absolute value notation.*

Example 14 *Express "x is within* δ *units of a" using absolute value notation.*

Example 15 *Determine whether each statement is True or False*

1)$|x|$ is a positive number. 2)$|-x| = x$ 3)If $x < 0$, then $|x| = -x$. 4)$d(m,m) = 0$ for any real number m. $d(a,b) = -d(b,a)$ for any real numbers a and b.