

1 P1 The Real Number System

Sets

We can place numbers with similar characteristics into sets. The following sets of numbers are used extensively in the study of algebra:

Natural Numbers $\{1, 2, 3, \dots\}$

Integers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational Numbers

Irrational Numbers

Real Numbers $\{\text{all rational or irrational numbers}\}$

Difference between Rational numbers and Irrational numbers

1) In the form $x = \frac{p}{q}$

If p and q are integers and $q \neq 0$, then x is rational. Otherwise x is irrational.

Example 1 Which of the following is rational and which is irrational? 1) $\frac{3}{4}$ 2) 5 3) $\sqrt[3]{2}$ 4) $-\sqrt{9}$ 5) $\frac{22}{7}$

2) In the Decimal Form

An irrational number neither terminates nor repeats.

Example 2 Which of the following is rational and which is irrational? 1) 0.75 2) $0.\overline{45}$ 3) 3.14 4) $0.272272227 \dots$ 5) π

The rational number 3.14 or the rational number $\frac{22}{7}$ are used as an approximation of the irrational number π .

Every real number is either a rational number or an irrational number.

Each member of a set is called an element of the set. A set A is a subset of a set B if every element of A is also an element of B , and we write $A \subseteq B$.

Example 3 The set of natural numbers is a subset of the set of integers.

Example 4 The set of integers is a subset of the set of rational numbers.

A **prime number** is a positive integer other than 1 that has no positive integer factors other than itself and 1.

A **composite number** is a positive integer greater than 1 that is not a prime number.

Example 5 Determine which of the following numbers are 1) integers 2) rational numbers 3) irrational numbers 4) real numbers 5) prime numbers 6) composite numbers

$-0.2, 0, 0.\overline{3}, \pi, 6, 7, 41, 51, 0.717717771 \dots, \sqrt{-9}$.

The empty set or null set is a set without any elements and it is denoted by ϕ .

Example 6 The set of numbers that are both prime and also composite is ϕ .

Sets are often written using set-builder notation.

Example 7 If $A = \{x^2 | x \text{ is an integer}\}$, then $A = \{0, 1, 4, 9, 16, 25, \dots\}$

Example 8 List the 4 smallest elements of the infinite set $\{y | y = 2x + 1, x \text{ is a natural number}\}$

The **intersection** of sets A and B , denoted by $A \cap B$, is the set of all elements belonging to both set A and set B .

The **union** of sets A and B , denoted by $A \cup B$, is the set of all elements belonging to set A , to set B , or to both.

Example 9 Given $A = \{1, 2, 3, 4, 5\}$, $B = \{x | x \text{ is an even number and } 0 \leq x < 8\}$, $C = \{x | x \text{ is a prime number less than } 10\}$

Find: 1) $A \cup (B \cap C)$ 2) $A \cap (B \cup C)$

Properties of Real Numbers

Addition of two real numbers a and b is designated by $a + b$. If $a + b = c$, then c is the **sum** and the real numbers a and b are called terms. The **additive inverse** of b is $-b$ and note that $a - b = a + (-b)$. **Subtraction** of two real numbers a and b is designated by $a - b$. If $a - b = c$, then c is called the **difference** of a and b .

Multiplication of two real numbers a and b is designated by ab or $a \cdot b$. If $ab = c$, then c is the **product** and the real numbers a and b are called factors. The **multiplicative inverse** or **reciprocal** of the nonzero number b is $\frac{1}{b}$ and note that $a \div b = a \left(\frac{1}{b}\right)$, $b \neq 0$. **Division** of two real numbers a and b is designated by $a \div b$ with $b \neq 0$. If $a \div b = c$, then c is called the **quotient** of a and b . We can represent $a \div b$ by the fractional notation $\frac{a}{b}$ where a is called the **numerator** and the nonzero b is called the **denominator**.

Properties of Real Numbers

1) Closure

The set is closed under any operation if we apply the operation to any elements in that set, then the result is still in the same set.

Example 10 Is the set $\{0, 1, -1\}$ closed under addition?

Example 11 Is the set $\{0, 1, -1\}$ closed under multiplication?

Example 12 Is the set of irrational numbers closed under addition?

Example 13 Is the set of irrational numbers closed under multiplication?

Example 14 Is the set of real numbers closed under addition?

Example 15 Is the set of real numbers closed under multiplication?

Example 16 Is the set of real numbers closed under division?

2) Commutative

$a + b = b + a$ illustrates the commutative property of addition where a and b are real numbers.

$ab = ba$ illustrates the commutative property of multiplication where a and b are real numbers.

Example 17 Since $a - b \neq b - a$, then the real numbers is not commutative under subtraction.

3) Associative

$(a + b) + c = a + (b + c)$ illustrates the associative property of addition where a and b are real numbers.

$(ab)c = a(bc)$ illustrates the associative property of multiplication where a and b are real numbers.

Example 18 $3 + (4 + 5) = 3 + (5 + 4)$ illustrates the property of addition.

4) Identity

There exists a unique real number 0 such that $a + 0 = 0 + a = a$.

There exists a unique real number 1 such that $a \cdot 1 = 1 \cdot a = a$.

5) Inverse

For each real number a , there is a unique real number $-a$ such that $a + (-a) = (-a) + a = 0$.

For each nonzero real number a , there is a unique real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

6) Distributive

$a(b + c) = ab + ac$ illustrates the distributive property.

Example 19 Identify the property of real numbers illustrated in each statement:

- 1) $a(bc) = a(cb)$ 2) $(2a)b = 2(ab)$ 3) $4(x + 3) = 4x + 12$ 4) $a + (-a) = 0$ 5) $(\frac{1}{2} \cdot 2)a = 1 \cdot a$ 6) $1 \cdot a = a$
7) $3(x + y) = 3(y + x)$ 8) $(\frac{1}{5})11$ is a real number. 9) $(a + 5b) + 7c = (5b + a) + 7c$.

An **equation** is a statement of equality between two numbers or two expressions

Properties of Equality Let a , b , and c be real numbers.

Reflexive $a = a$

Symmetric If $a = b$, then $b = a$.

Transitive If $a = b$ and $b = c$, then $a = c$.

Substitution If $a = b$, then a may be replaced by b in any expression that involves a .

Example 20 Identify the property of Equality illustrated in each statement:

- 1) If $2x + 3y = 7$, and $x^2 + y^2 = 7$, then $2x + 3y = x^2 + y^2$. 2) $5(x + y) = 5(x + y)$ 3) If $3a + b = c$, then $c = 3a + b$. 4) If $4a - 1 = 7b$ and $7b = 5c + 2$, then $4a - 1 = 5c + 2$.

Properties of Fractions

For all fractions $\frac{a}{b}$ and $\frac{c}{d}$, where $b \neq 0$ and $d \neq 0$.

Equality $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Equivalent Fractions $\frac{a}{b} = \frac{ac}{bc}$, $c \neq 0$.

Addition $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Subtraction $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

Multiplication $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Division $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, $c \neq 0$.

Sign $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Properties of Zero

- 1) For $a \neq 0$, $\frac{0}{a} = 0$.
- 2) $\frac{a}{0}$ is undefined.
- 3) 0 is neither a positive nor a negative number.
- 4) 0 is an even number.

Example 21 Use the properties of fractions to perform the indicated operations:

- 1) $\frac{4z}{7} + \frac{3z}{2}$
- 2) $\frac{7x}{5} \div \frac{2x}{15}$
- 3) $\frac{1}{x} - \frac{1}{y}$

Example 22 State the multiplicative inverse of $7\frac{3}{8}$.

Example 23 State the multiplicative inverse of -0.75 .

Example 24 Classify each statement as True or False.

- 1) $\frac{a}{0}$ is the multiplicative inverse of $\frac{0}{a}$.
- 2) If $a - b = 7$, then $7 = b - a$.
- 3) The sum of two composite numbers is a composite number.
- 4) The product of two composite numbers is a composite number.
- 5) 1 is the only positive integer that is not prime and not composite.
- 6) Any integer is also a rational number.
- 7) The number zero is both rational and irrational.
- 8) Each real number is either even or odd.
- 9) The sum of two irrational numbers is an irrational number.
- 10) Every rational number has a multiplicative inverse.
- 11) $\pi = \frac{22}{7}$.

Example 25 Which properties were used to prove:

- 1) $[a + (-a)] \cdot b = b \cdot 0$.
- 2) $ax + ay = (x + y)a$

Example 26 If $A = \{-\sqrt{9}, \frac{\pi}{2}, \frac{-3}{16}, 0.67, \sqrt{8}, -\sqrt{-100}\}$, then determine which of the elements of A are 1) Integers 2) Natural numbers 3) Rational numbers 4) Irrational numbers 5) Real numbers.