

1 Section 9.1 Systems of Linear Equations in Two Variables

An equation of the form $Ax + By = C$ is a **linear equation in two variables**. A **solution** of a linear equation in two variables is an ordered pair (x, y) that makes the equation a true statement.

Example 1 $(-2, 3)$ is a solution of the equation $2x + 3y = 5$.

A **system of equations** is two or more equations considered together.

Example 2 $\begin{cases} 2x + 3y = 4 \\ 3x - 2y = 7 \end{cases}$ is a linear system of equations in two variables.

A **solution** of a system of equations in two variables is an ordered pair that is a solution of both equations.

The graphs of two linear equations in two variables can intersect at a single point, be the same line, or be parallel. When the graphs intersect at a single point or are the same line, the system is called a **consistent** system of equations. The system is called an **independent** system of equations when the lines intersect at exactly one point. The system is called a **dependent** system of equations when the equations represent the same line. In this case, the system has infinite number of solutions. When the graphs of the two equations are parallel lines, the system is called **inconsistent** and has no solution.

To solve a system of linear equations, we can use the **substitution method** or the **elimination method**.

Example 3 Solve $\begin{cases} 5x + 2y = -4 \\ y = -3x \end{cases}$ by the substitution method.

Example 4 Solve $\begin{cases} 3x + 2y = -4 \\ 9x + 6y = -8 \end{cases}$ by the substitution method.

Example 5 Solve $\begin{cases} 8x - 4y = 16 \\ 2x - y = 4 \end{cases}$ by the substitution method.

There is always more than one way to describe the ordered pairs when writing the solution of a dependent system of equations. For the last example either the ordered pairs $(c, 2c - 4)$ or the ordered pairs $(\frac{1}{2}b + 2, b)$ would generate all the solutions of the system of the equations.

Two systems of equations are **equivalent** if each system has exactly the same solution.

Example 6 $\begin{cases} 3x + 5y = 9 \\ 2x - 3y = -13 \end{cases}$ and $\begin{cases} x = -2 \\ y = 3 \end{cases}$ are equivalent system of equations.

Operations that produce Equivalent System of Equations

1. Interchange any two equations.
2. Replace an equation with a nonzero multiple of that equation.
3. Replace an equation with the sum of that equation and a nonzero constant multiple of another equation in the system.

Example 7 Solve $\begin{cases} 8x + 5y = 9 \\ 3x - 2y = -16 \end{cases}$ by the elimination method.

Example 8 Solve $\begin{cases} x - 2y = 2 \\ 3x - 6y = 6 \end{cases}$ by the elimination method.

If an equation of the system of equations is replaced by a false equation, then the system of equations has no solution.

Exercise 9 The system of equations $\begin{cases} x + y = 4 \\ 0 = 5 \end{cases}$ has no solution.

Example 10 Solve $\begin{cases} 3\sqrt{2}x - 4\sqrt{3}y = -6 \\ 2\sqrt{2}x + 3\sqrt{3}y = 13 \end{cases}$ by the elimination method.

Example 11 A chemist wants to make 50 milliliters of a 16% acid solution. How many milliliters each of a 13% acid solution and an 18% acid solution should be mixed to produce the desired solution.

Example 12 Use the fact that if $z_1 = a_1 + b_1i$ and $z_2 = a_2 + b_2i$ are two complex numbers, then $z_1 = z_2$ if and only if $a_1 = a_2$ and $b_1 = b_2$ to solve for x and y .

1. $(2 + i)x + (3 - i)y = 7$
2. $\begin{cases} 2x + 5y = 11 + 3i \\ 3x + y = 10 - 2i \end{cases}$