1 Section 8.2 Ellipses

An **ellipse** is the set of all points in the plane, the sum of whose distances from two fixed points (**Foci**) is a positive constant.

The graph of an ellipse has two sets of symmetry. The longer axis is called the **major axis**. The foci of the ellipse are on the major axis. The shorter axis is called the **minor axis**. The semi axes are one-half the axes in length. the center of the ellipse is the midpoint of the major axis. The endpoints of the major axis are the **vertices** of the ellipse.

The **standard form** of the equation of an ellipse with the center at the origin and major axis on the x-axis is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$.

The length of the major axis is $2a$.The length of the minor axis is $2b$. The coordinates of the vertices are $(a, 0)$ and $(-a, 0)$, and the coordinates of the foci are $(c, 0)$ and $(-c, 0)$, where $c^2 = a^2 - b^2$.

The **standard form** of the equation of an ellipse with the center at the origin and major axis on the y-axis is given by $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $a > b$.

The length of the major axis is $2a$.The length of the minor axis is $2b$. The coordinates of the vertices are $(0, a)$ and $(0, -a)$, and the coordinates of the foci are $(0, c)$ and $(0, -c)$, where $c^2 = a^2 - b^2$.

Example 1 *Find the vertices and foci of the ellipse given by the equation* $\frac{x^2}{64} + \frac{y^2}{4} = 1$. *Sketch the graph.*

Example 2 *Find the equation of the ellipse in standard form with center at the origin, foci* $(0, 3)$, $(0, -3)$ *and major axis of length* 10.

Standard Forms of the Equation of an Ellipse with Center at (h, k)

Major Axis Parallel to the x-axis

The standard form of the equation of an ellipse with the center at (h, k) and major axis parallel to the x-axis is given by $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad a > b$. The length of the major axis is $2a$. The length of the minor axis is $2b$. The coordinates of the vertices are $(h + a, k)$ and $(h - a, k)$, and the coordinates of the foci are $(h + c, k)$ and $(h - c, k)$, where $c^2 = a^2 - b^2$.

Major Axis Parallel to the y-axis

The standard form of the equation of an ellipse with the center at (h, k) and major axis parallel to the y-axis is given by $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \qquad a > b$. The length of the major axis is $2a$. The length of the minor axis is $2b$. The coordinates of the vertices are $(h, k + a)$ and $(h, k - a)$, and the coordinates of the foci are $(h, k + c)$ and $(h, k - c)$, where $c^2 = a^2 - b^2$.

Example 3 *Find the vertices and foci of the ellipse* $4x^2+9y^2-8x+36y+4 = 0$. *Sketch the graph.*

Example 4 *Find the standard form of the equation of the ellipse with center at* $(4, -2)$*, foci* $(4, 1)$ *and* $(4, -5)$*, and minor axis of length* 10.

Example 5 *Find the standard form of the equation of the ellipse with vertices* $(-7,-1)$ *and* $(5,-1)$, *foci at* $(-5,-1)$ *and* $(3,-1)$.

Example 6 *Find the standard form of the equation of the ellipse with foci at* $(-1,1)$ *and* $(-1,7)$ *that passes through the point* $\left(\frac{3}{4},1\right)$.

Eccentricity of an Ellipse

The eccentricity e of an ellipse is the ratio of c to a, where c is the distance from the center to a focus and a is one-half the length of the major axis. That is $e = \frac{c}{a}$. The eccentricity of an ellipse is a measure of its "roundness."

Because $c < a$, for an ellipse, $0 < e < 1$. When $e \approx 0$, the graph is almost a circle. When $e \approx 1$, the graph is long and thin.

Example 7 *Find the eccentricity of the ellipse given by* $8x^2 + 9y^2 = 18$.

Example 8 *Use the eccentricity of each ellipse to find its equation in standard form: 1)Eccentricity* $\frac{1}{4}$, *foci at* $(-2,4)$ *and* $(-2,-2)$. *2)Eccentricity* $\frac{3}{5}$, *major axis of length* 15 *on the x-axis, center at* $(0,0)$.