1 Section 8.1 Parabolas

The graph of a parabola, circle, ellipse, or hyperbola can be formed by the intersection of a plane and a cone. They referred to them as **conic sections**.

A **parabola** is the set of points in the plane that are equidistant from a fixed line (the **Directrix**) and a fixed point (the **Focus**) not on the directrix.

The line that passes through the focus and is perpendicular to the directrix is called the **axis of symmetry** of the parabola. The midpoint of the line segment between the focus and directrix on the axis of symmetry is the **vertex** of the parabola.

The **standard form** of the equation of a parabola with vertex $(0,0)$ and the y-axis as its axis of symmetry is $x^2 = 4py$. The focus is $(0,p)$ and the equation of the directrix is $y = -p$. Note that this parabola is a function.

Note that the parabola opens up when $p > 0$ and opens down when $p < 0$.

The **standard form** of the equation of a parabola with vertex $(0,0)$ and the x-axis as its axis of symmetry is $y^2 = 4px$. The focus is $(p,0)$ and the equation of the directrix is $x = -p$. Note that this parabola is not a function.

Note that the parabola opens to the right when $p > 0$ and opens to the left when $p < 0$.

Example 1 *Find the focus and the directrix of the parabola given by the equation $x = 4y^2$.*

Example 2 *Find the equation of the parabola in standard form with vertex at the origin and focus at $(0, -4)$.*

Standard Forms of the Equations of a Parabola with Vertex at (h, k)

Vertical Axis of Symmetry

The standard form of the equation of the parabola with vertex $V(h, k)$ and a vertical axis of symmetry is $(x - h)^2 = 4p(y - k)$.

The focus is $(h, k + p)$, and the equation of the directrix is $y = k - p$.

Horizontal Axis of Symmetry

The standard form of the equation of the parabola with vertex $V(h, k)$ and a horizontal axis of symmetry is $(y - k)^2 = 4p(x - h)$.

The focus is $(h + p, k)$, and the equation of the directrix is $x = h - p$.

Example 3 *$(x - 3)^2 = 2(y + 1)$ defines a parabola. Determine each of the following: 1) the vertex   2)the value of p 3)the focus   4)the directrix 5)the axis of symmetry.*

Example 4 *Find the equation of the directrix and the coordinates of the vertex and the focus of the parabola given by the equation $3x + 2y^2 + 8y - 4 = 0$.*

Example 5 *Find the equation in standard form of the parabola with directrix $x = -1$ and focus $(3, 2)$.*

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Example 6 *Find the equation in standard form of the parabola that has vertex* $(-4, 1)$, *has its axis of symmetry parallel to the y-axis, and passes through the point* $(-2, 2)$.