

# 1 Section 6.3 Double and Half Angle Identities

## Double-Angle Identities

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha & \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1 \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \end{aligned}$$

**Example 1** Write  $\cos^2 3x - \sin^2 3x$  as a single trigonometric function.

**Example 2** Write  $4 \sin 5\theta \cos 5\theta$  as a single trigonometric function.

**Example 3** For an angle  $\beta$  in Quadrant II,  $\cos \beta = -\frac{4}{5}$ , find  $\cos 2\beta$ .

**Example 4** Verify the identity: 1)  $\csc 2\alpha = \frac{1}{2}(\tan \alpha + \cot \alpha)$     2)  $\cos 4x = 1 - 8 \cos^2 x + 8 \cos^4 x$

$$3) \cos^3 x - \sin^3 x = (\cos x - \sin x) \left(1 + \frac{1}{2} \sin 2x\right).$$

## Half-Angle Identities

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \quad \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \quad \tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

Note that the sign of the radical is determined in which the terminal side of angle  $\frac{\alpha}{2}$  lies.

**Example 5** Verify the identity: 1)  $\sin^2 \frac{x}{2} = \frac{\sin^2 x}{2} + \frac{\cos x(\cos x - 1)}{2}$     2)  $\sin \frac{x}{2} - \cos \frac{x}{2} = \sqrt{1 - \sin x}$ ,  $0^\circ \leq x \leq 90^\circ$

**Example 6** If  $\alpha$  is an angle in the given quadrant, in which quadrant is the angle  $\frac{\alpha}{2}$ ?

1) Quadrant I    2) Quadrant II    3) Quadrant III    4) Quadrant IV

**Example 7** Verify the identity  $\tan \frac{x}{2} = \frac{\tan x}{\sec x + \sin x \csc x}$

**Example 8** Write the following in terms of a single trigonometric function:

$$1) \cos^2 3\alpha - \sin^2 3\alpha \quad 2) \frac{2 \tan 3\alpha}{1 - \tan^2 3\alpha} \quad 3) \frac{3 \tan 3\alpha}{4 - 4 \tan^2 3\alpha}$$

**Example 9** Find the exact value of the following: 1)  $\cos 157.5^\circ$     2)  $\sin \frac{3\pi}{8}$

**Example 10** Find the exact value of the sine, cosine, and tangent of  $\frac{\alpha}{2}$  given the following information:

$$\cos \alpha = -\frac{8}{17}, \alpha \text{ in Quadrant III.}$$

**Example 11** If  $x + y = 180^\circ$ , verify that  $\sin(x - y) = -\sin 2x$ .