

1 Section 6.2 Sum, Difference, and Cofunction Identities

Sum and Difference Identities

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Cofunction Identities

$$\sin(90^\circ - \theta) = \cos \theta \quad \cos(90^\circ - \theta) = \sin \theta \quad \tan(90^\circ - \theta) = \cot \theta$$

$$\cot(90^\circ - \theta) = \tan \theta \quad \sec(90^\circ - \theta) = \csc \theta \quad \csc(90^\circ - \theta) = \sec \theta$$

If θ is in radian measure, we can replace 90° with $\frac{\pi}{2}$.

Example 1 Write each of the following in terms of a single trigonometric function

1. $\cos 10x \cos 9x + \sin 10x \sin 9x$

2. $\sin 7x \cos 3x - \cos 7x \sin 3x$

3. $\frac{\tan 5\beta - \tan 3\beta}{1 - \tan 5\beta \tan 3\beta}$

4. $\sin x \sin 5x - \cos x \cos 5x$

Example 2 Given $\cos \alpha = -\frac{15}{17}$ for α in Quadrant III and $\tan \beta = -\frac{5}{12}$ for β in Quadrant II, find $\sin(\alpha - \beta)$.

Example 3 Verify the identity $\sin(\frac{3\pi}{2} + \beta) = -\cos \beta$.

Example 4 True or False: 1) For any integer k , $\cos k\pi = (-1)^k$. 2) For any integer k , $\sin k\pi = 0$. 3) For any integer k , $\cos \frac{k\pi}{2} = 0$. 4) $\cos \frac{k\pi}{2} = \begin{cases} 0 & \text{when } k \text{ is odd} \\ (-1)^{\frac{k}{2}} & \text{when } k \text{ is even} \end{cases}$.

Example 5 Verify the identity $\cos(\theta + 2k\pi) = \cos \theta$, where k is any integer.

Example 6 Find the exact value of $\sin 165^\circ$.

Example 7 Verify the identity 1) $\sin(\alpha - \beta) - \sin(\alpha + \beta) = -2 \cos \alpha \sin \beta$. 2) $\cos 3x = 4 \cos^3 x - 3 \cos x$

3) $\sin(\frac{\pi}{2} + \alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \cos \beta$

Example 8 Find the exact value of the following: 1) $\cos(45^\circ - 30^\circ)$ 2) $\tan(\frac{\pi}{6} + \frac{\pi}{4})$ 3) $\cos \frac{\pi}{12} \cos \frac{\pi}{4} - \sin \frac{\pi}{12} \sin \frac{\pi}{4}$

4) $\frac{\tan \frac{\pi}{6} + \tan \frac{\pi}{3}}{1 - \tan \frac{\pi}{6} \tan \frac{\pi}{3}}$