

# 1 Section 5.6 Graphs of the Other Trigonometric Functions

## The graph of the Tangent Function

**Example 1** Graph  $y = \tan x$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

The lines  $x = \frac{\pi}{2}$  and  $x = -\frac{\pi}{2}$  are vertical asymptotes for the graph of  $y = \tan x$ .

The period of  $y = \tan x$  is  $\pi$ .

There is no amplitude for the tangent function because it is unbounded.

**Example 2** Graph  $y = -3 \tan x$ .

The period of  $y = \tan bx$  is  $\frac{\pi}{|b|}$ .

One cycle of the graph of  $y = \tan bx$  is completed on  $(-\frac{\pi}{2b}, \frac{\pi}{2b})$ .

**Example 3** Graph  $y = \frac{1}{2} \tan 2x$  on  $(-\frac{\pi}{2}, \frac{\pi}{2})$ .

**Example 4** Find an equation of the following graph.

**Example 5** For what values of  $x$  is  $y = \tan x$  undefined?

One cycle of the graph of  $y = a \tan bx$  for both  $a$  and  $b$  positive has the following.

1. The period is  $\frac{\pi}{b}$ .
2.  $x = 0$  is a zero.
3. The graph passes through  $(-\frac{\pi}{4b}, -a)$  and  $(\frac{\pi}{4b}, a)$ .
4. The vertical asymptotes are  $x = -\frac{\pi}{2}$  and  $x = \frac{\pi}{2}$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

## The Graph of the Cotangent Function

**Example 6** Graph  $y = \cot x$  for  $0 < x < \pi$ .

The line  $x = 0$  and  $x = \pi$  are vertical asymptotes for the graph of  $y = \cot x$ .

The period of  $y = \cot x$  is  $\pi$ .

There is no amplitude for the cotangent function.

The period of  $y = \cot bx$  is  $\frac{\pi}{|b|}$ .

One cycle of the graph of  $y = \cot bx$  is completed on the interval  $(0, \frac{\pi}{b})$ .

**Example 7** Graph  $y = 2 \cot \pi x$ .

One cycle of the graph of  $y = a \cot bx$  for both  $a$  and  $b$  positive has the following.

1. The period is  $\frac{\pi}{b}$ .
2.  $x = \frac{\pi}{2b}$  is a zero.
3. The graph passes through  $(\frac{\pi}{4b}, a)$  and  $(\frac{3\pi}{4b}, -a)$ .
4. The vertical asymptotes are  $x = 0$  and  $x = \frac{\pi}{b}$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

**Example 8** Find an equation of the following graph

**Example 9** For what values of  $x$  is  $y = \cot x$  undefined?

### The Graph of the Cosecant Function

$$\csc x = \frac{1}{\sin x}.$$

The graph of  $y = \csc x$  has vertical asymptotes at  $x = n\pi$  where  $n$  is an integer.

**Example 10** Graph  $y = 2 \csc \frac{\pi}{2}x$ .

**Example 11** Find an equation of the following graph

One cycle of the graph of  $y = a \csc bx$  for both  $a$  and  $b$  positive has the following.

1. The period is  $\frac{2\pi}{b}$ .
2. The vertical asymptotes are the zeros of  $y = a \sin bx$ .
3. The graph passes through  $(\frac{\pi}{2b}, a)$  and  $(\frac{3\pi}{2b}, -a)$ .
4. There are no zeros for the graph of  $y = a \csc bx$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

### The Graph of the Secant Function

$$\sec x = \frac{1}{\cos x}.$$

The graph of  $y = \sec x$  has vertical asymptotes at  $x = \frac{\pi}{2} + n\pi$  where  $n$  is an integer.

**Example 12** Graph  $y = -3 \sec \frac{1}{2}x$ .

**Example 13** Find an equation of the following graph

One cycle of the graph of  $y = a \sec bx$  for both  $a$  and  $b$  positive has the following.

1. The period is  $\frac{2\pi}{b}$ .
2. The vertical asymptotes are the zeros of  $y = a \cos bx$ .

3. The graph passes through  $(0, a)$ ,  $(\frac{\pi}{b}, -a)$  and  $(\frac{2\pi}{b}, a)$ .
4. There are no zeros for the graph of  $y = a \sec bx$ .
5. If  $a < 0$ , the graph is reflected across the x-axis.

**Example 14** Graph 1)  $y = |\cot x|$       2)  $y = |\csc x|$

**Example 15** Graph the following:      1)  $y = 3 \cot(2x + \pi)$       2)  $y = 3 \csc(x + 2\pi)$ .

**Example 16** Find an equation of the following graphs

## 2 Section 5.7 Graphing Techniques

**Example 17** Graph  $y = x + \sin x$  on  $[-2\pi, 2\pi]$

**Example 18** Graph  $y = \cos x - \sin x$  for  $-2\pi \leq x \leq 2\pi$